BMT 922 : Neural Signal Analysis and Modeling

Basics of Neurophysics

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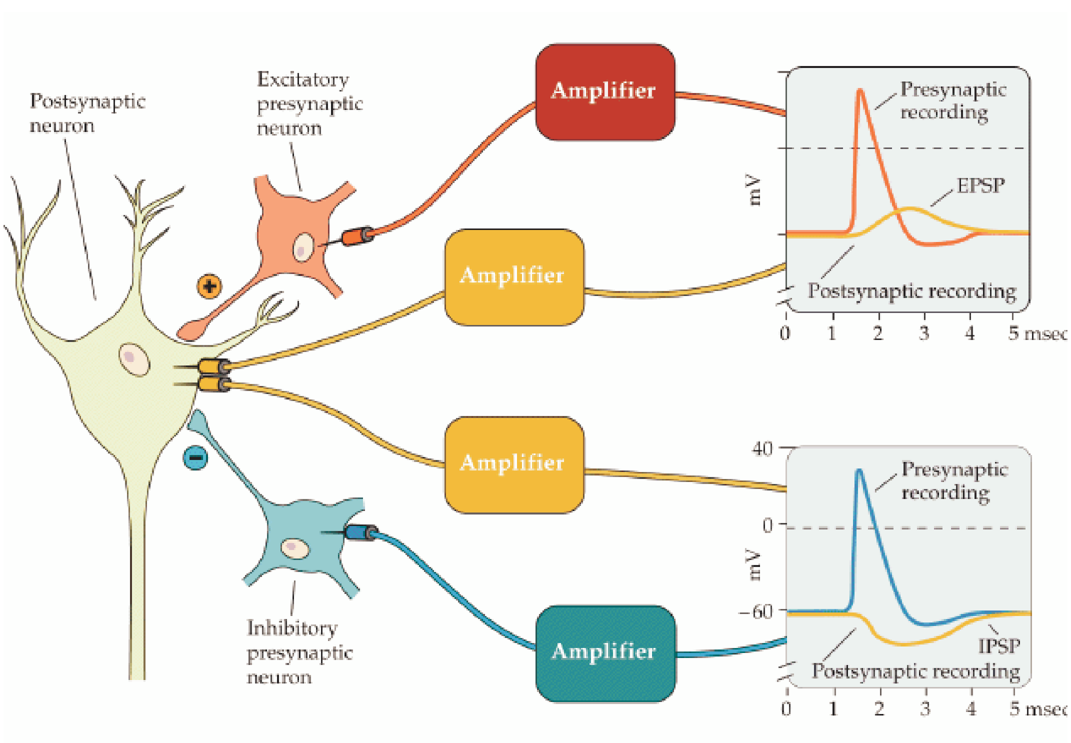
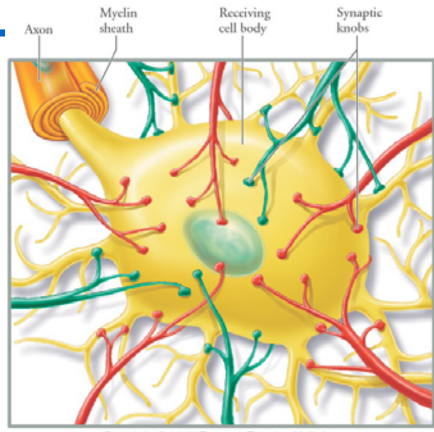
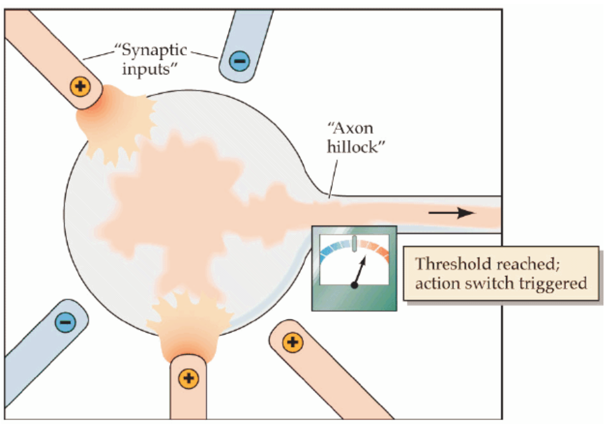
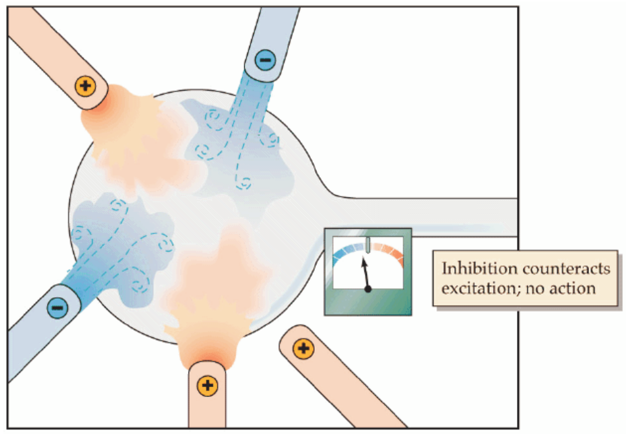


Spatial integration (superposition)







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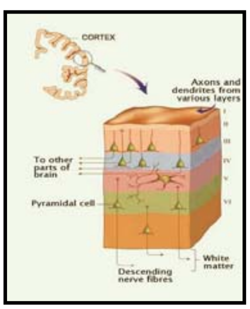
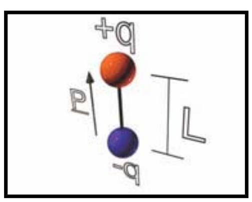
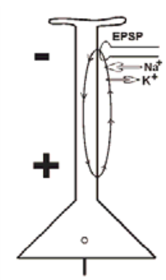
Pyramidal Cells





EIPSP

- Have pyramid shaped cell bodies
- They are large and have long, spiny dendrites
- Axonal processes run vertically at 90° to the surface of the cortex
- Pyramidal neurones are projection neurones whose contacts are made in the grey matter



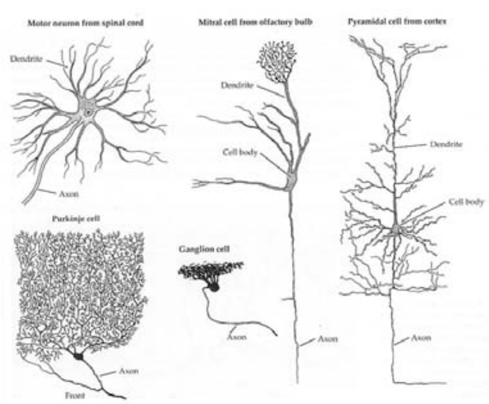
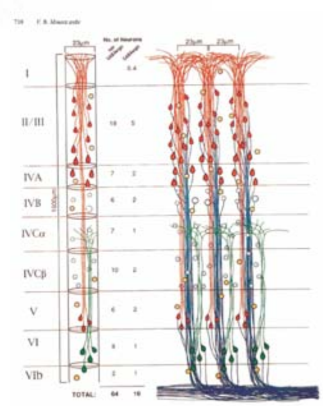


FIGURE 1.4 Shapes and Sizes of Neurons. Neurons have branches (the dendrites) on which other neurons form synapses, and axons that in turn make connections with other neurons. The motor neuron, drawn by Deiters in 1869, was dissected from a mammalian spinal cord. The other cells, stained by the Golgi method, were drawn by Ramón y Cajal. The pyramidal cell is from the cortex of a mouse, the mitral cell from the olfactory bulb (a relay station in the pathway concerned with smell) of a rat, the Purkinje cell from human cerebellum, and the ganglion cell from mammalian retina (animal not specified). (After Ramón y Cajal, 1995.)



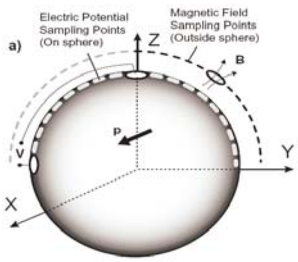
Forward: Volume Conductor Models

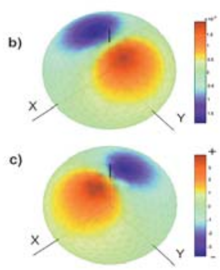
- **Volume conductor models** estimate electric potentials at the measurement locations given a source distribution (well posed).

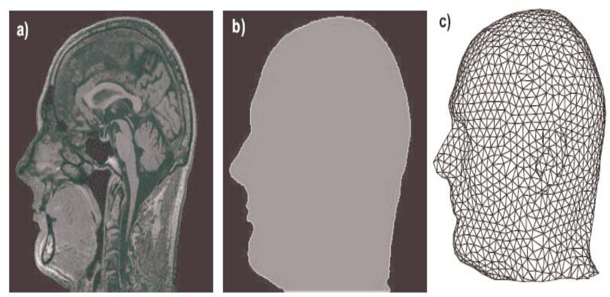
  a) Single-Sphere Model: Easy to compute but does not consider the smearing effect of the skull.

  b) Multiple layer-Sphere Models: More accurate representation of the brain layer components (brain, skull and scalp) but does not account for the inhomogeneity of the human brain. (Multi channel recording is possible) (Zhang, 1994,1999)

  c) Realistic head models: Use of real human head models, volume currents cannot be disregarded as in Sphere Models. (Sensor fitted sphere and 3D interpolation scheme algorithms, Elmer et. al.) BEM, FEM and FDM (Hallez et. al., 2005)







electromagnetic basics

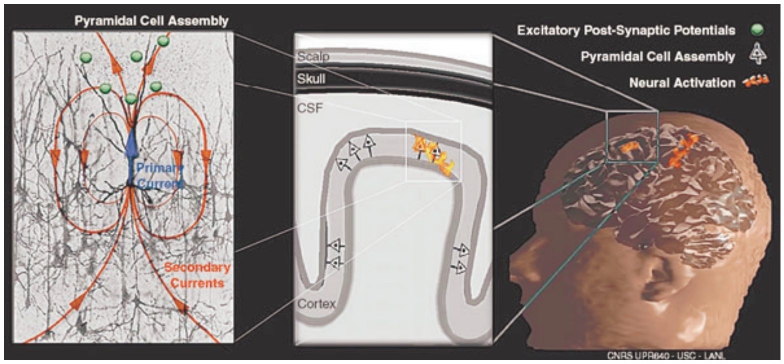
Maxwell Equations:

$$
\begin{aligned}
\nabla \times \mathbf{E}(\mathbf{x},t) &= -\frac{\partial}{\partial t}\mathbf{B}(\mathbf{x},t)\\
\nabla \times \mathbf{H}(\mathbf{x},t) &= \mathbf{J}(\mathbf{x},t)\\
\nabla \cdot \mathbf{B}(\mathbf{x},t) &= 0\\
\nabla \cdot \mathbf{D}(\mathbf{x},t) &= \rho(\mathbf{x},t)
\end{aligned}
$$

$$\mathbf{D}(\mathbf{x},t) = \epsilon(\mathbf{x})\mathbf{E}(\mathbf{x},t),\ \mathbf{B}(\mathbf{x},t) = \mu(\mathbf{x})\mathbf{H}(\mathbf{x},t)$$

"Physiological" Simplifications:
(quasistatic, homogenous $\mu = \mu_0$)

$$
\begin{aligned}
\nabla \times \mathbf{E}(\mathbf{x}) &= 0\\
\nabla \times \mathbf{B}(\mathbf{x}) &= \mu\mathbf{J}(\mathbf{x})\\
\nabla \cdot \mathbf{B}(\mathbf{x}) &= 0\\
\nabla \cdot \mathbf{D}(\mathbf{x}) &= \rho(\mathbf{x})
\end{aligned}
$$



primary currents:

Physical (Feynman) dipole $J_F^p : \mathbb{R}^3 \mapsto \mathbb{R}$

$$J_F^p(\mathbf{x}) = \Upsilon\left(\delta(\mathbf{x}-\mathbf{x}_q) - \delta(\mathbf{x}-\mathbf{x}_s)\right)$$

(monopolar strength: $\Upsilon \in \mathbb{R}$; source: $\mathbf{x}_q \in \mathbb{R}^3$; sink: $\mathbf{x}_s \in \mathbb{R}^3$)

Mathematical dipole $J^p : \mathbb{R}^3 \mapsto \mathbb{R}$

$$J^p(\mathbf{x}) = \nabla \cdot \mathbf{J}_M^p(\mathbf{x}) = \nabla \cdot \mathbf{m}\delta(\mathbf{x}-\mathbf{x}_0)$$

(dipole moment: $\mathbf{m} \in \mathbb{R}^3$; position: $\mathbf{x}_0 \in \mathbb{R}^3$)

primary + secondary currents:

$$\mathbf{J} = \mathbf{J}^p + \sigma E = \mathbf{J}^p - \sigma\nabla\phi$$

Poisson's equation

Poisson's equation

$$-\nabla \cdot (\sigma\nabla\phi) = -\nabla \cdot \mathbf{J}^p = -J^p \text{ in } \Omega$$

For this equation, Neumann boundary conditions on the head surface $\Gamma = \partial\Omega$ are given as

$$\langle \sigma\nabla\phi, \mathbf{n}\rangle_2 = 0,$$

and additionally a reference electrode condition can be given as

$$\phi_{\mathbf{ref}} = 0,$$

together with a compatibility condition provided by Gauss's theorem

$$\int_\Omega J^p d\Omega = 0.$$

variational formulation

Here we assume $\mathbf{J}^p \in L^2(\Omega)$. By means of the bilinear form $a(\cdot,\cdot)$ and the functional $l(\cdot)$, the Poisson equation with Neumann boundary conditions can be defined as follows:

$$a(\phi, v) := \int_\Omega \nabla\phi\sigma\nabla v d\Omega$$

$$l(v) = \langle l, v\rangle := -\int_\Omega \mathbf{J}^p v d\Omega$$

where $\phi \in H^1(\Omega)$ and $\mathbb{V} = H^1(\Omega)$. With this definition the formulation of the problem is as follows: We are searching for $\phi \in \mathbb{V}$ so that

$$a(\phi, v) = \langle l, v\rangle \text{ for all } v \in \mathbb{V}$$

For the numerical solution we choose a finite dimensional subspace $\mathbb{V}_h \subset \mathbb{V}$ with dimension $\mathbb{V}_h = N_h$ and basis functions $\psi_1, ..., \psi_{N_h}$. Here h denotes the average mesh size. We now approximate ϕ using a Ritz-Galerkin approach:

$$a(\phi_h, v_h) = \langle l, v_h\rangle \text{ for all } v_h \in \mathbb{V}_h.$$

For each coefficient vector $\phi_h \in \mathbb{V}_h$ with $\mathbb{V}_h := \mathbb{R}^{N_h}$, we define $P : V_h \longrightarrow \mathbb{V}_h$ with

$$\phi_h(\mathbf{x}) = P\phi_h := \sum_{i=1}^{N_h} \phi_h^{[i]}\psi_i(\mathbf{x})$$

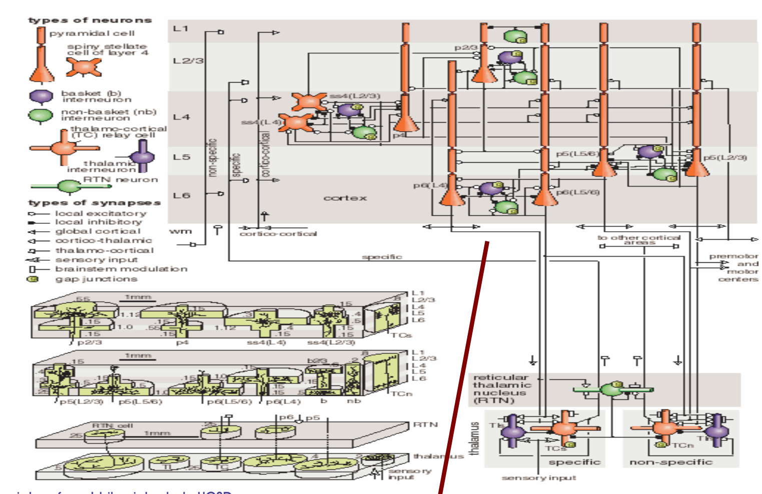
where $\phi_h^{[i]}$ denotes the i^{th} component of the vector ϕ_h. The discrete variational problem can be transformed into a system of liner equations

$$K_h\phi_h = J_h$$

with

$$K_h^{[ij]} := a(\psi_j, \psi_i) \text{ for all } 1 \le i, j \le N_h,$$
$$J_h^{[i]} := l(\psi_i) \text{ for all } 1 \le i \le N_h.$$

Neural Fields & Corticothalamic Feedback

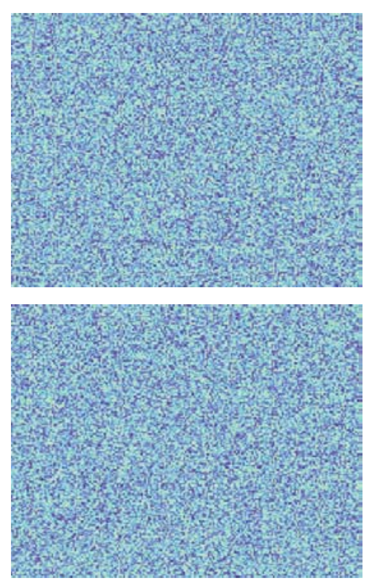


picture from Izhikevich et al., UCSD

$$\psi(x,t) = \int_\Gamma dX f(x,X)\cdot S[\psi(X, t - \frac{|x-X|}{v}) + p(\phi_a(\Pi, G_i, \vartheta(k))],$$

Poisson's equation for the neural field:

$$-\nabla\cdot(\sigma(\cdot)\nabla\phi(\cdot)) = -\nabla\cdot(\psi(\cdot,t)\mathbf{n}(\cdot)) \text{ in } \Omega$$



DJ Strauss et al. Corticofugal Modulation due to Listening Effort. Cog Neurodyn. In Press

Amari equation:

$$\tau\frac{\partial u(x,t)}{\partial t} - u(x,t) = \int_{\mathbb{R}^m} w(x,y) f(u(y,t))\, dy, \quad x \in \mathbb{R}^m, t > 0$$

$$\tau\frac{du_i(t)}{dt} + u_i(t) = \sum_{j=1}^N w_{ij} f(u_j(t))$$ for discrete networks

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corticothalamic transfer function

$$\psi(x,t) = \int_{\Gamma} dX\, f(x,X) \cdot S[\psi(X, t - \frac{|x-X|}{v}) + p(\phi_a(\Pi, G_i, \vartheta(k))],$$

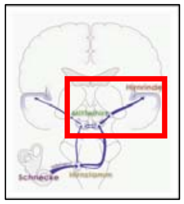
Poisson's equation for the neural field:

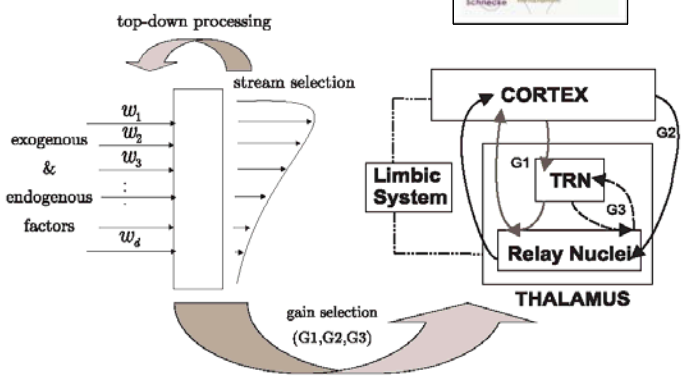
$$-\nabla \cdot (\sigma(\cdot)\nabla\phi(\cdot)) = -\nabla \cdot (\psi(\cdot,t)\mathbf{n}(\cdot)) \text{ in } \Omega$$

$$S[V_a(\boldsymbol{r},t)] = \frac{Q_{\max}}{1+\exp\{-[V_a(\boldsymbol{r},t)-\theta]/\sigma'\}}$$

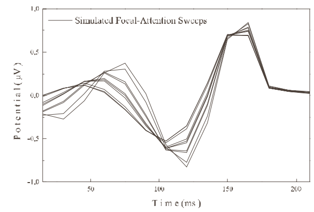
$$\left(\frac{1}{\gamma_a^2}\frac{\partial^2}{\partial t^2} + \frac{2}{\gamma_a}\frac{\partial}{\partial t} + 1 - r_a^2\nabla^2\right)\phi_a(\boldsymbol{r},t) = S[V_a(\boldsymbol{r},t)]$$

$$\frac{\phi_{e,i}(k,w)}{A_n(k,w)} = \xi(\Pi, G1, G2, G3, L, t_d, \vartheta(k))$$





C Trenado, L Haab, DJ Strauss.
Corticothalamic feedback dynamics for neural correlates of auditory selective attention. *IEEE Trans Neural Syst Rehabil Eng.* pp. 17:46-52, 2009



neurofunctional mapping to the hearing path

Fig. 1. Left: Simplified probabilistic model of the auditory selection by top–down processes. Right: Corticothalamic feedback dynamics in our model as represented by three different gains: Gain **G1:** The auditory cortex projects indirectly to thalamic reticular nucleus (TRN) by means of axon–collaterals of corticothalamic projections. Additionally the TRN receives inhibitory input from dorsal thalamic nuclei. Thus, the TRN provides an inhibitory influence on the specific thalamus cores, namely the medial geniculate body (MGB) in the case of auditory evoked potentials. The target of TRN projections are the ventral and the medial subnuclei of MGB. The ventral subnucleus (VMGB) is specific for auditory processing, while the medial subnucleus (MMGB) receives also information from non–auditory pathways. The VMGB projects to anterior auditory field (AAF), the posterior auditory field (PAF) and the primary (A1) auditory cortex, the MMGB projects to the ipsilateral parts of the primary (A1), and the secondary (A2) auditory cortex, and to the ipsilateral posterior PAF and the anterior AAF auditory fields. Gain **G2:** The auditory cortex projects directly to all the subnuclei of MGB, namely VMGB, MMGB, and the dorsal geniculate body (DMGB), which also gets informational input from earlier stages of the auditory pathway. Back projection to the cortex occurs as described above plus efferent projections from the dorsal subnucleus (DMGB) to the auditory cortex. Gain **G3:** As described above the auditory cortex projects indirectly to the TRN by means of axon–collaterals of corticothalamic projections. The TRN has no efferent fibres projecting towards the auditory cortex, but is part of a thalamocortical feedback loop. The TRN receives additional input by axon–collaterals of thalamocortical projections. Due to its inhibitory influence on specific thalamus cores (i.e. MGB) the TRN can directly regulate information flow from thalamus to cortical areas.

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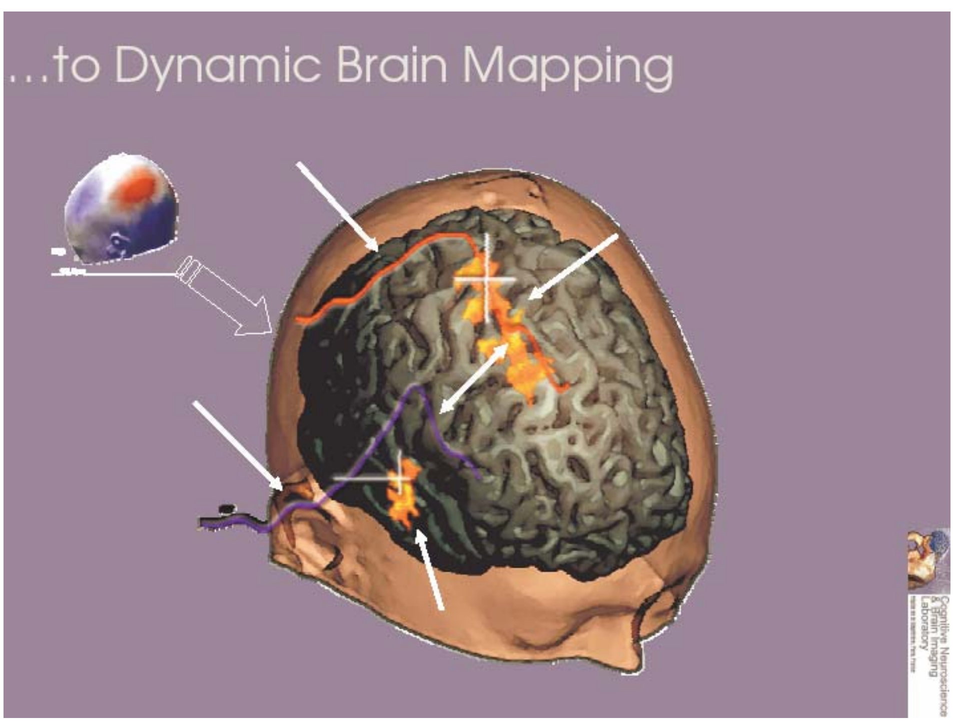
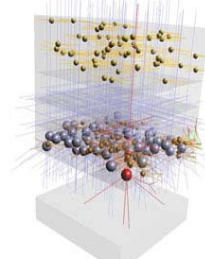
inverse EEG Source Localization



Neural field models

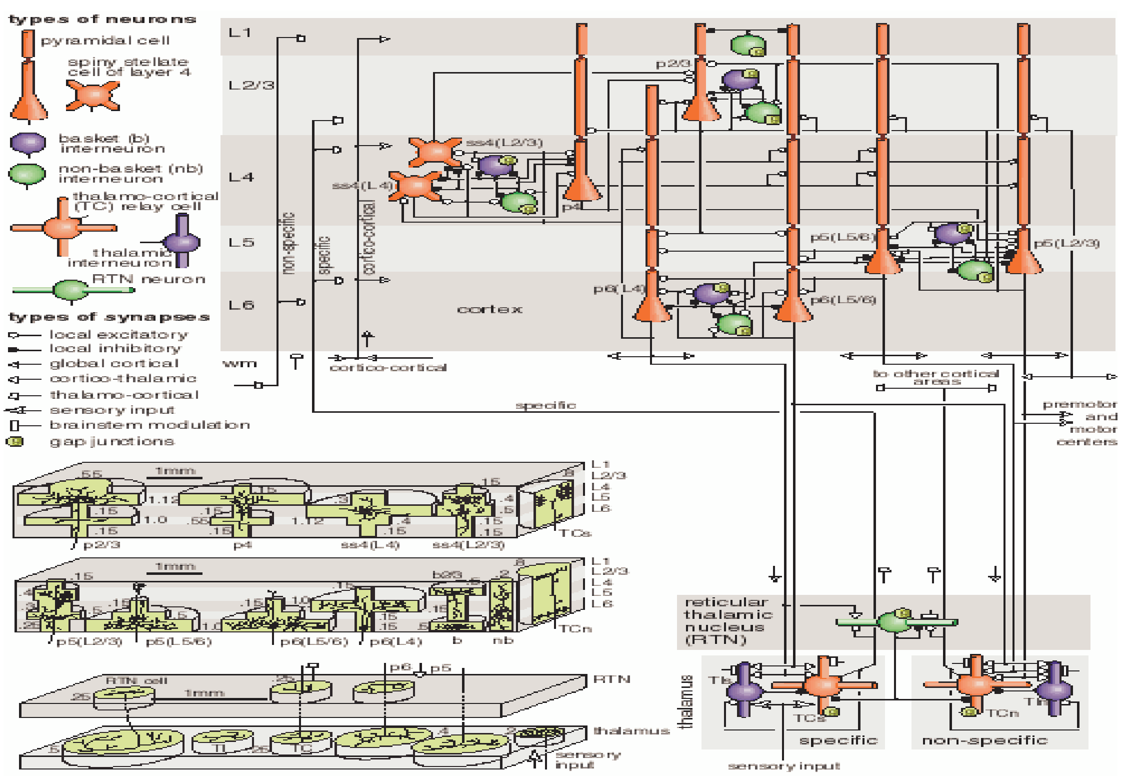


Models that consider neuronal-assemblies immersed in global fields of synaptic action to describe the spatio- temporal behavior of the human brain.

Two distinct approaches based on brain organization:

1. The cortex treated as a homogenous sheet of interconnected neurons that generate waves of propagating excitatory activity.
   (Wilson, 1973; Wilson & Cowan, 1973; Nunez, 1974; van Rotterdam et al., 1982; Jirsa & Haken, 1996; Wright & Liley, 1996; Robinson et al., 1997, 2001; Liley et al., 2002).

2. Neural field models on cortical colums
   On a scale of approximately 0.1–0.3 mm, the neocortex is organized into patches of densely interconnected neurons known as cortical columns. The neocortical activity can thus be modeled as an array of weakly coupled dynamical subsystems.



Corticothalamic loops

Simplified outline of corticothalamic loops