Systems Neuroscience
& Neurotechnology Unit

ERP Image Processing

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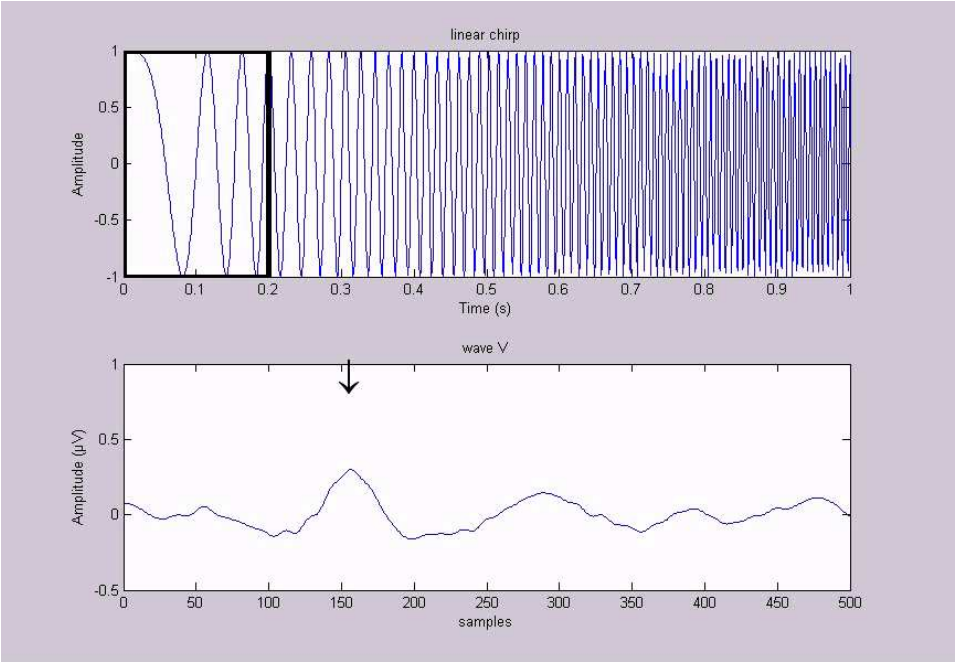
Single Sweep Processing

- Why do I need to process the aquired data?

- technical artifacts and noise:
  - power source (the famous 50Hz noise (60Hz in the USA))
  - subject movement
  - …
- Physiological variability:
  - background EEG
  - habituation (?)
  - endogeneous processes
  - vigilance (?)
  - attention (?)
  - changing a solution strategy
  - frequency specific stimulation (?)
  - plasticity
- Do I know the *exact* signal? Why?

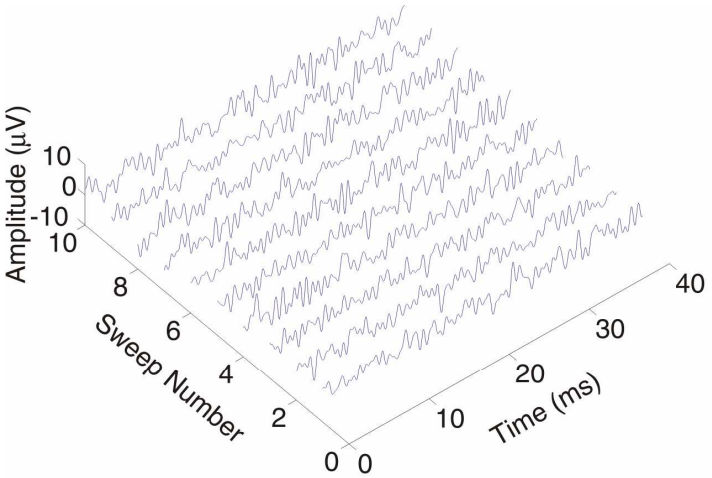


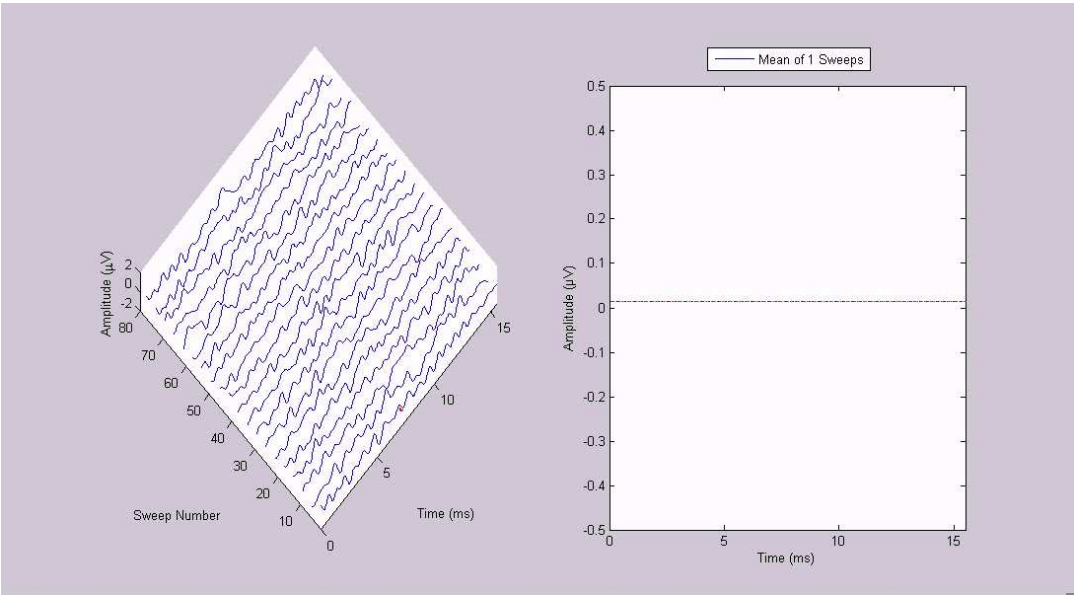
Any solutions or ideas?

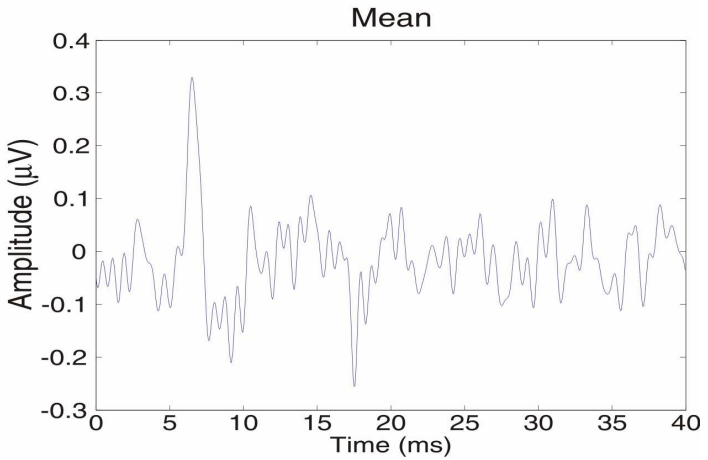
Averaging: simple, but…

- detecting small signals in large background noise

$$\bar{s}(t) = \frac{1}{N}\sum_{i=1}^{N} s_i(t) \qquad SNR \sim \sqrt{N}$$





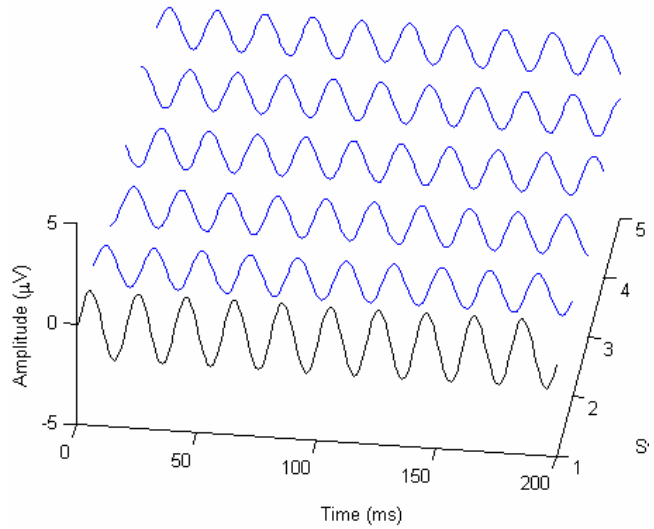


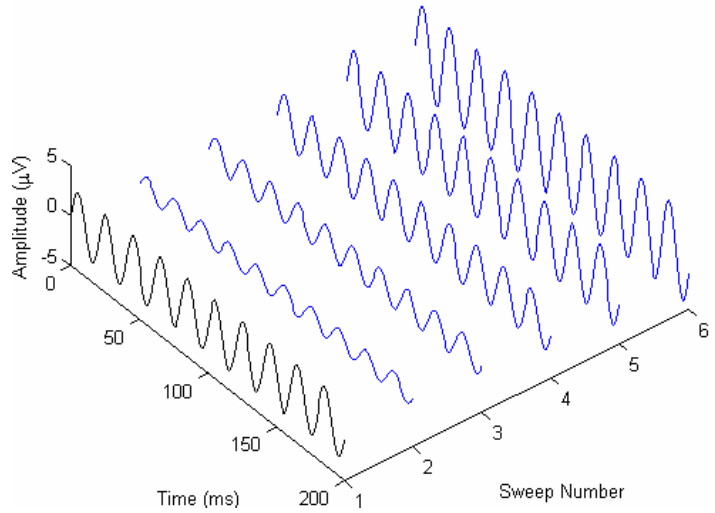
Averaging: simple, but…

Any drawbacks of the averaging method?

- morphological changes of EP components *during* measurements due to non-stational endogenous conditions (attention, vigilance)
- changing a solution strategy during measurements

The final mean won‘t show these changes!





Single-Sweep Images

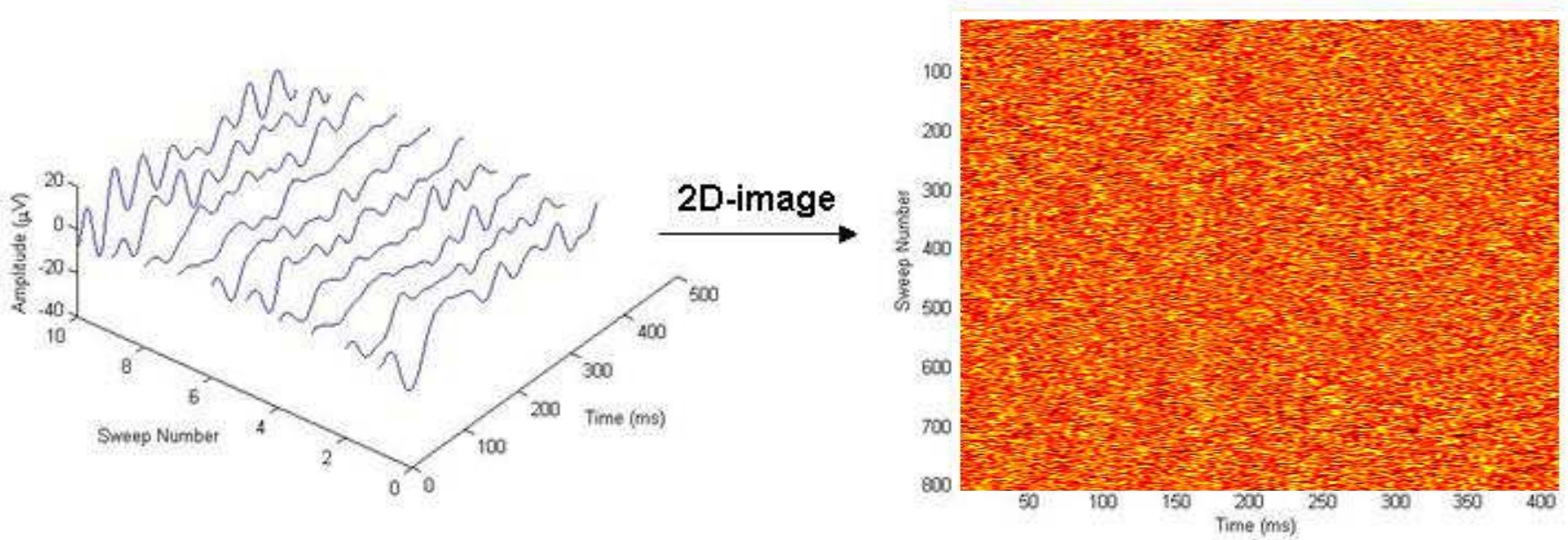


Image Processing Methods

Image Processing Methods – Gaussian smoothing

- convolution of a Gaussian function

$$G(x,y) = A \cdot e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}$$

$$(f * g)(\tau) = \int f(t) \cdot g(t-\tau)\, dt$$

- behaves like a low-pass filter
- weighted average between pixels
- increasing $\sigma \rightarrow$ smoother image

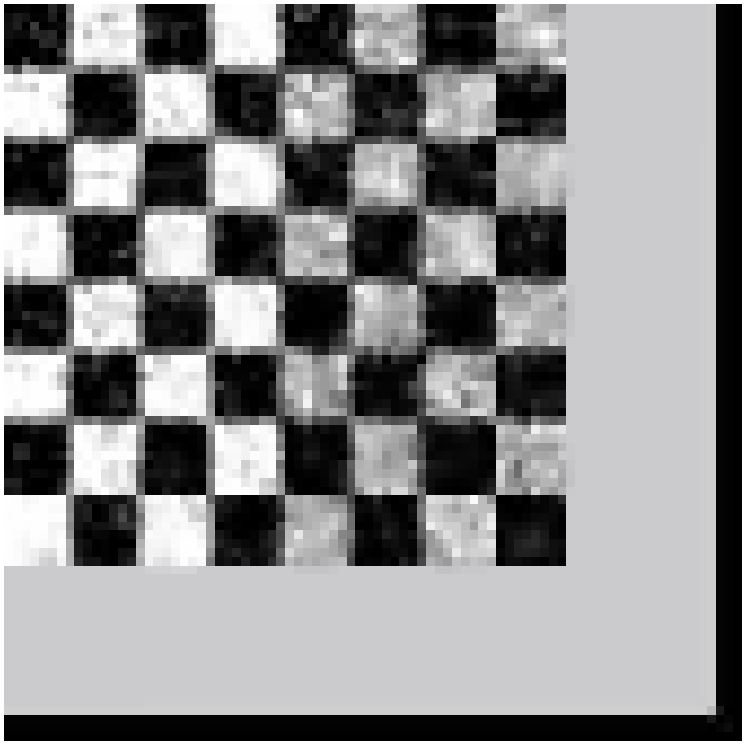


Image Processing Methods – Gaussian smoothing

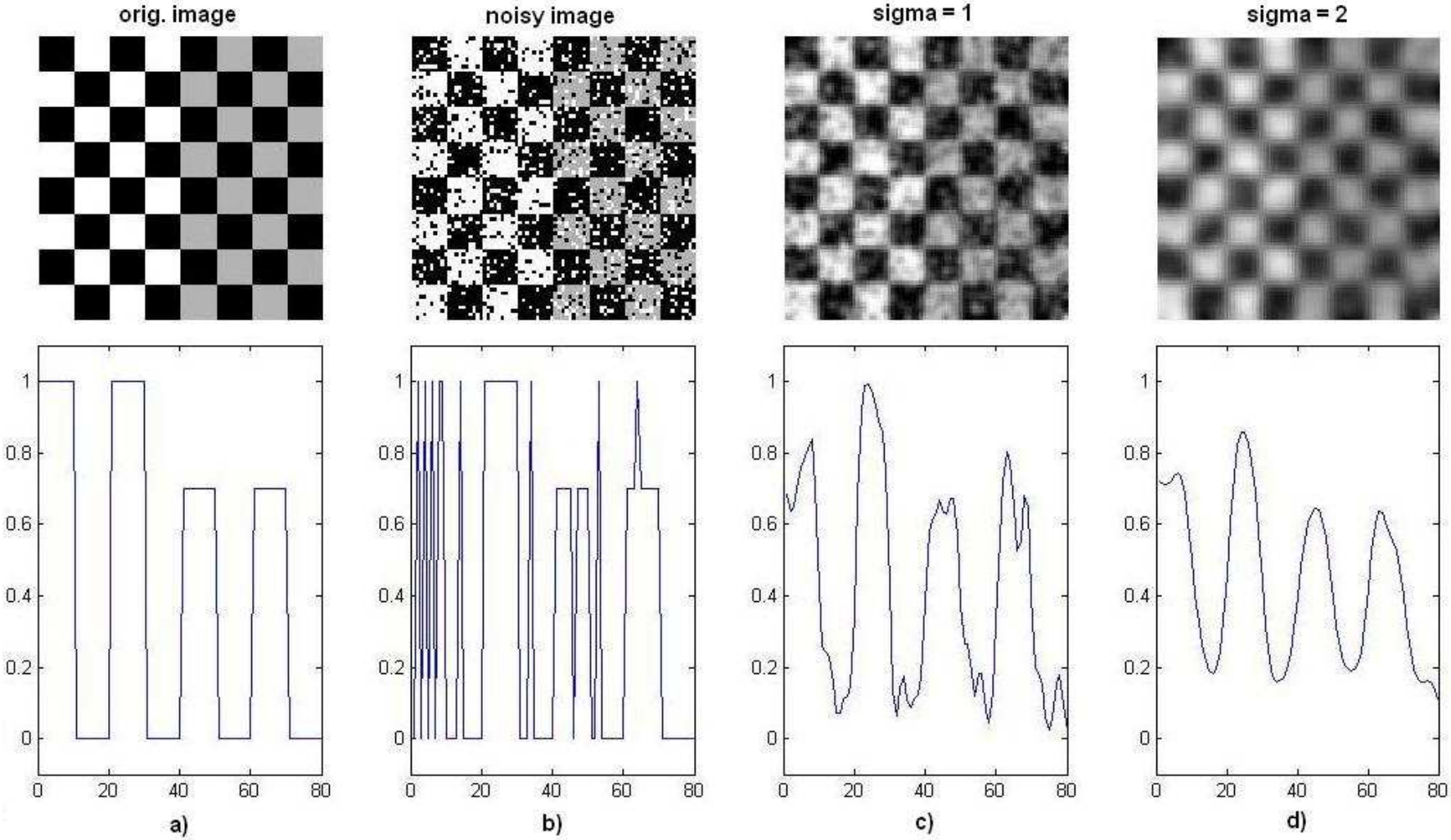
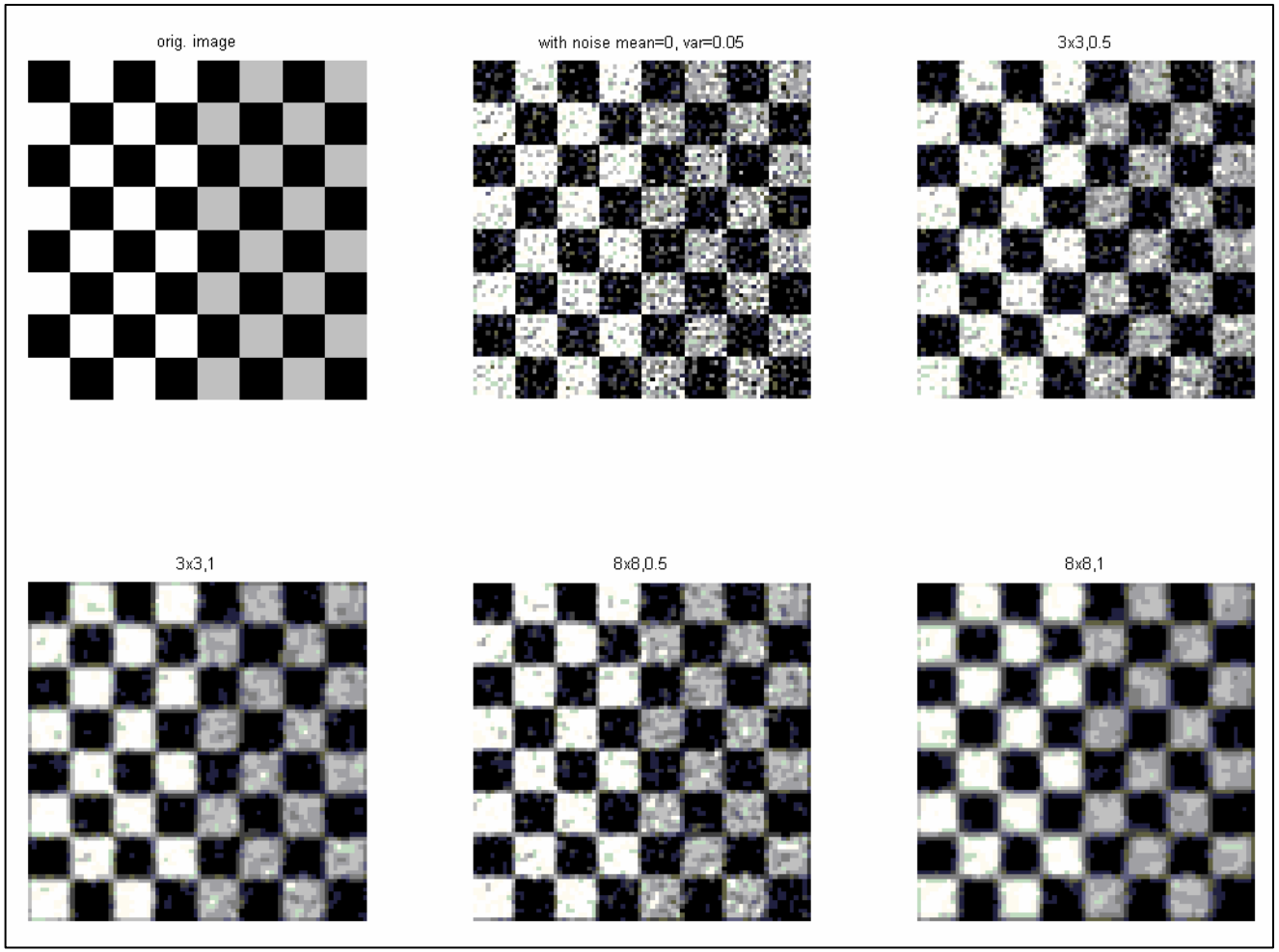


Image Processing Methods – Gaussian smoothing



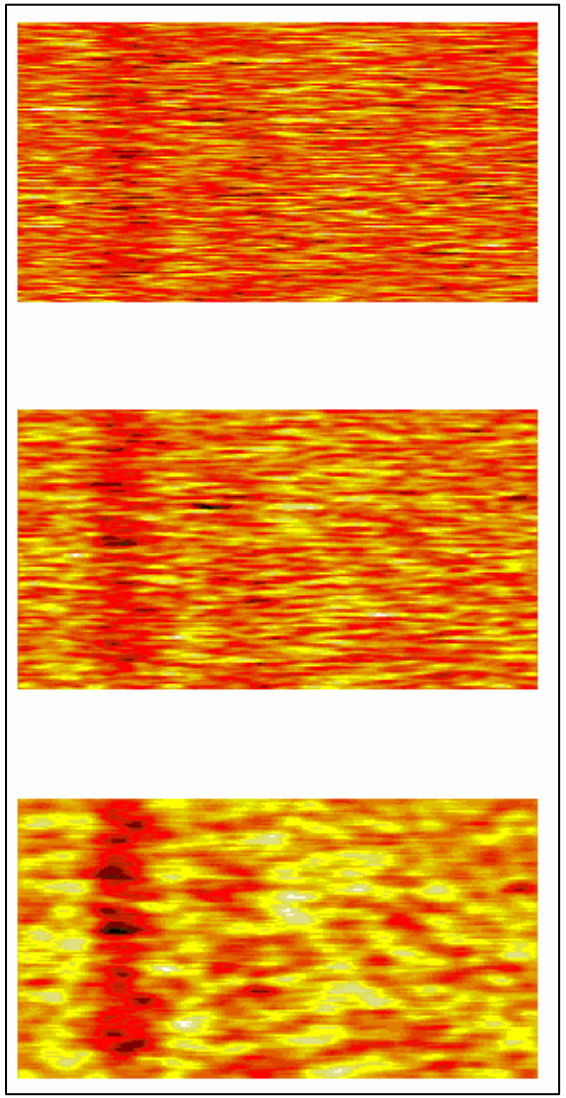


Image Processing Methods – Diffusion Filter

- based on two elemental equations:
  - Fick's law: $j = -D\nabla u$
  - continuity equation: $\partial_t u = -div(j)$
  - combined: $\partial_t u = div(D\nabla u)$
- diffusivity D can be chosen as $D = g\left(|\nabla u|^2\right)$
- where e.g.: $g = \frac{1}{\left(1+\left(x^2 / \lambda^2\right)\right)}$
- kind of edge detecting (increasing x^2 → decreasing D and hence decreasing the flux j)
- drawback: noise remains on edges

Image Processing Methods – Diffusion Filter

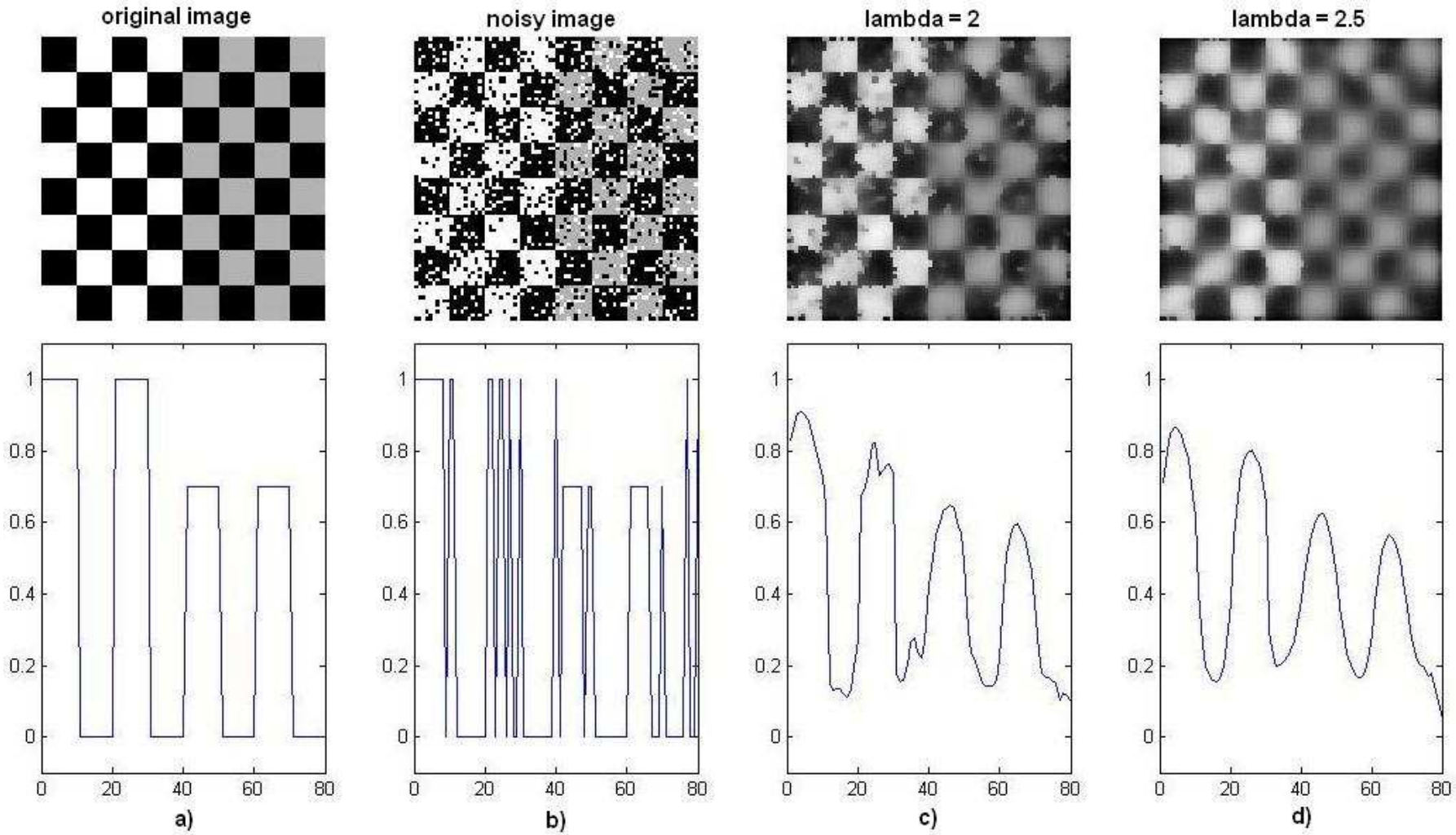
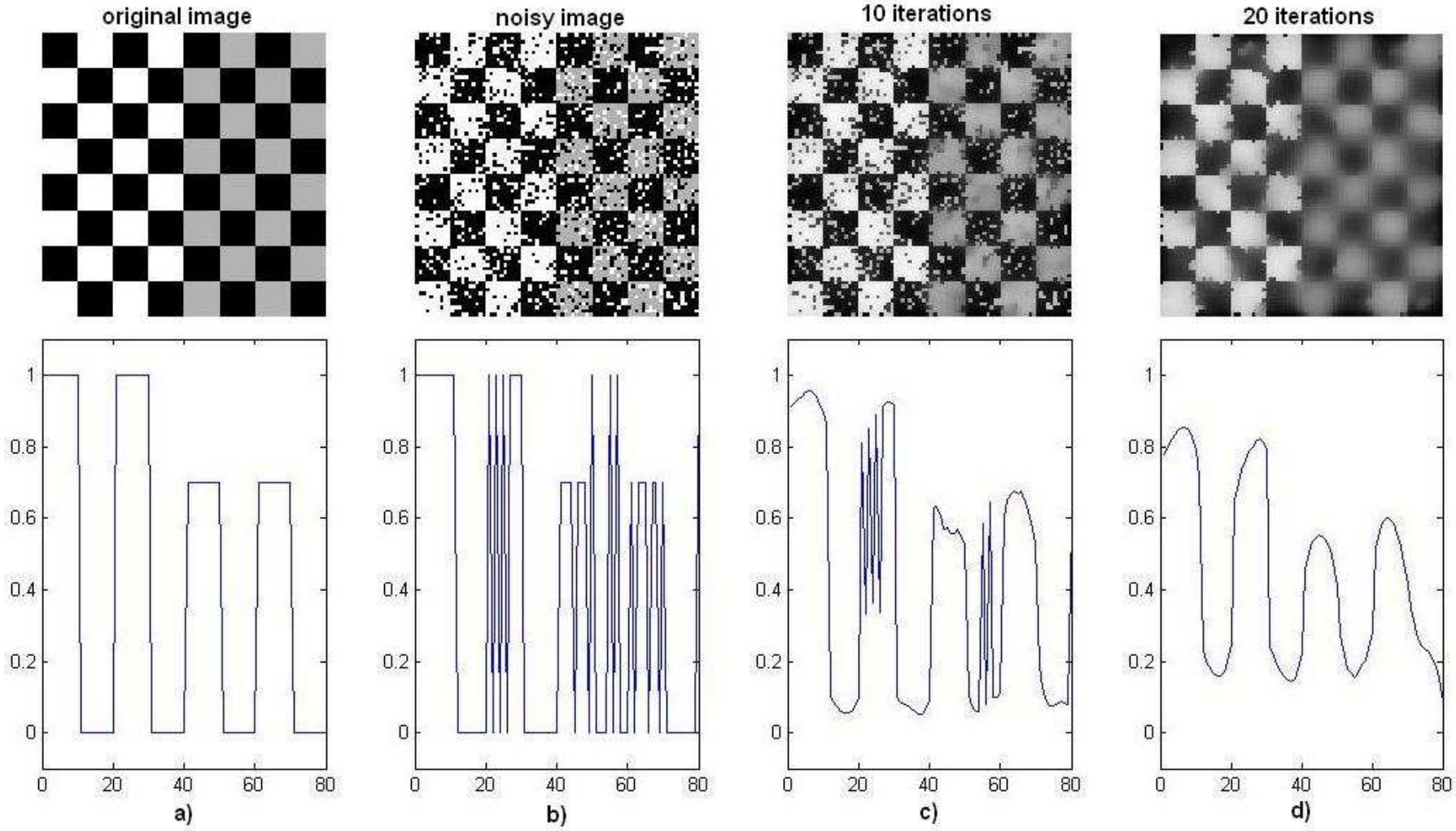
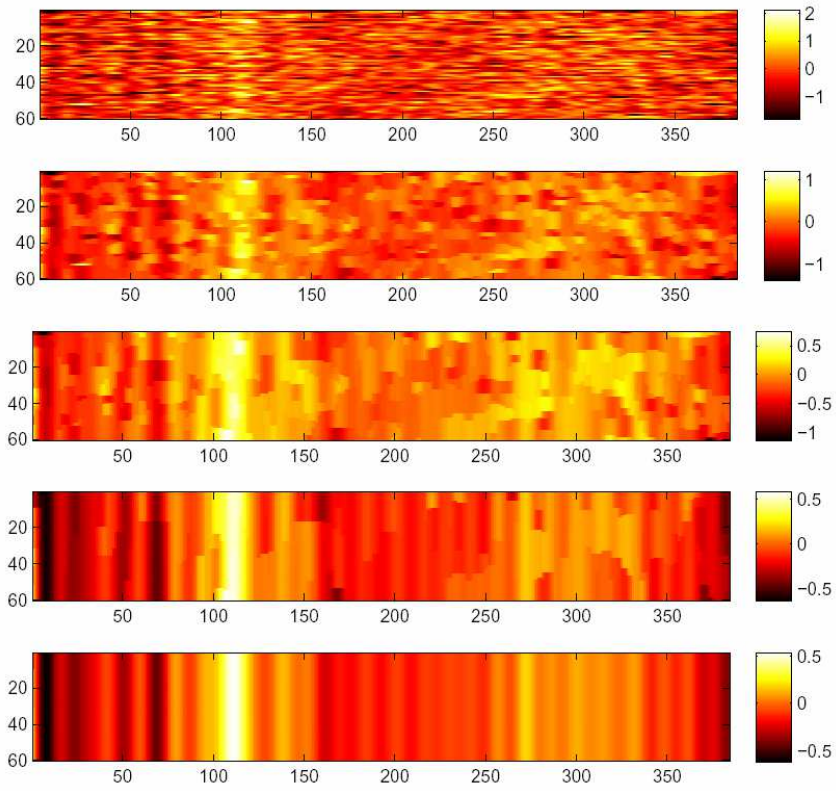


Image Processing Methods – Diffusion Filter

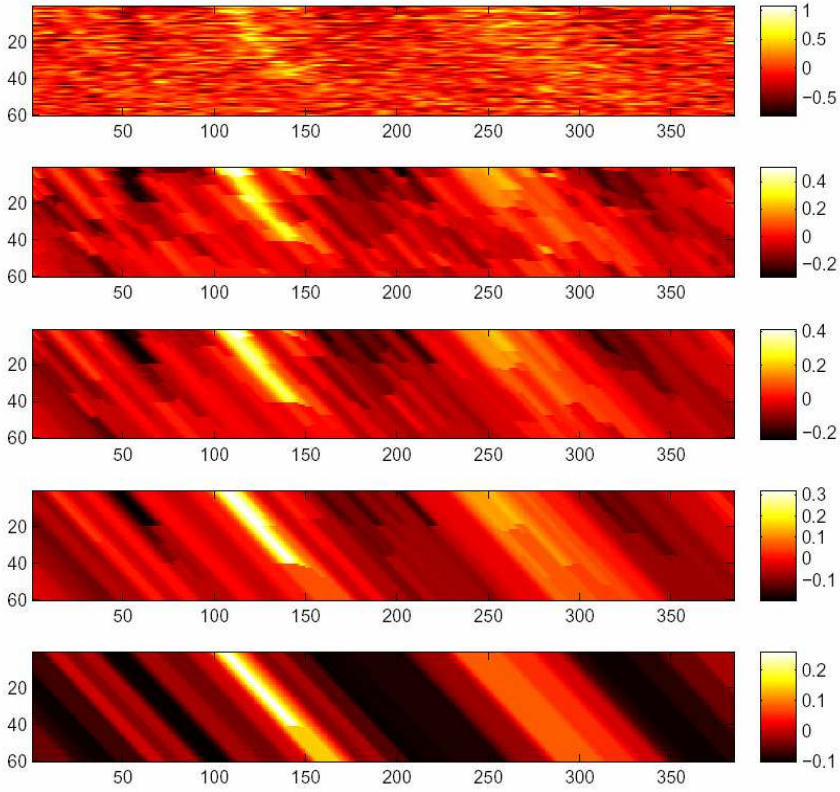


Anisotropic Diffusion

$$\hat{u} = \arg\min_u \frac{1}{2} \int (f(x) - u(x))^2 \, dx + \lambda \int |\langle v, \nabla u(x) \rangle| dx$$



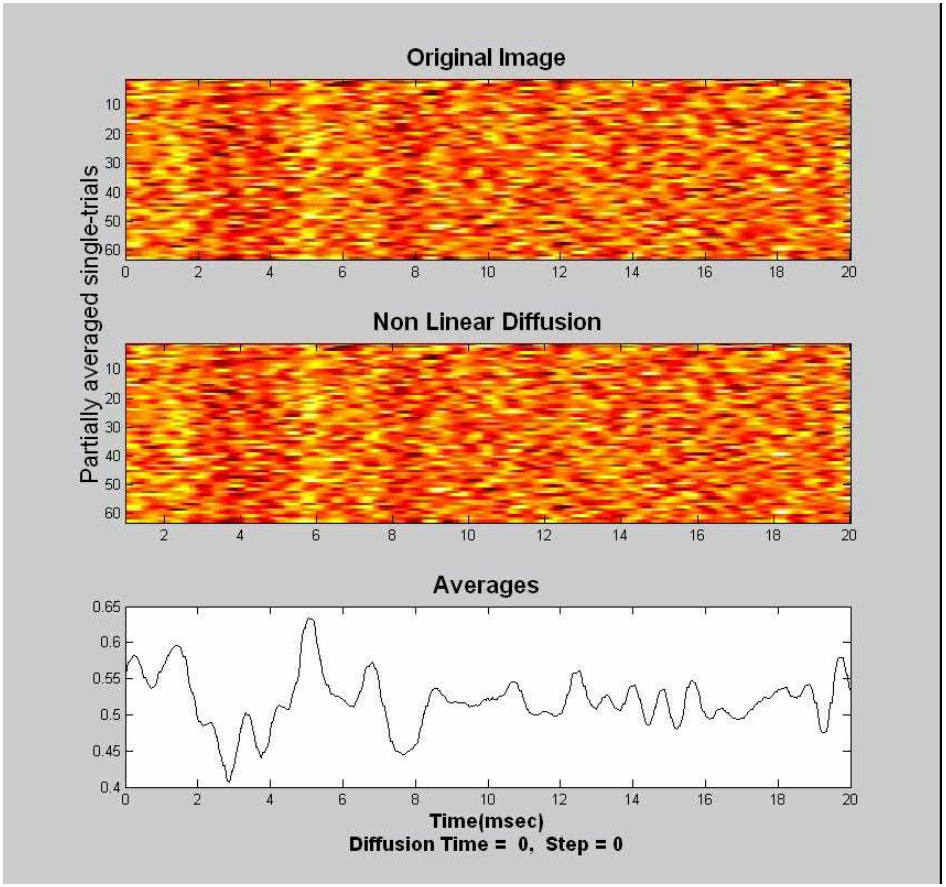
ABR (fixed stim. level)



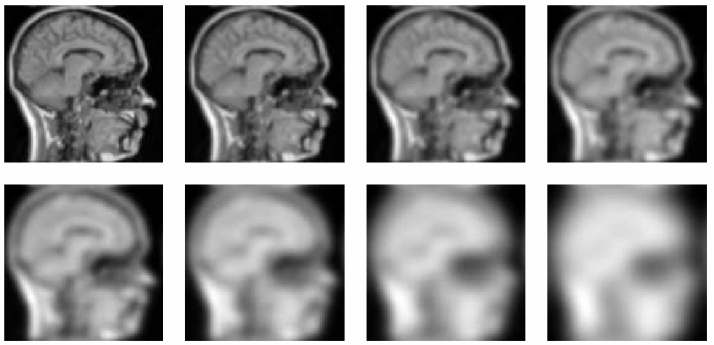
ABR (var. stim. level)

Image Processing Methods – Diffusion Filter

$$\partial_t u = \mathrm{div}\left(D(J) \cdot \nabla u\right)$$



Gaussian Scale Space



"blurring"

↓

nonlinear model

↓

anisotropic (tensor) diffusion

(inclusion of *a priori* information in ERP Matrix Representations)

I. Mustaffa, K. Schwerdtfeger, C. Trenado, D. J. Strauss.
Evoked Related Potential Denoising Using Nonlinear Diffusion Filtering of Single-Trial Matrix Representations.
J Neurosci Meth, pp. 512-520, 2009.

Singular Value Decomposition (SVD)

$$M \in \Re^{m \times n} \longrightarrow M = U \Sigma V^T$$

$$\Sigma \in \Re^{m \times n} \qquad \boxed{U \in \Re^{m \times m}} \qquad \boxed{V \in \Re^{n \times n}}$$

$$\Sigma = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_r \end{pmatrix} = diag\left(\{\sigma_i\}_{1 \le i \le r}\right)$$

σ = Singular values

r = Rank = $\min(m, n)$

$$\boxed{\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_r \ge 0}$$

Image Processing Methods: Regularization

- Regularization problem:

$$\arg\min_{X} \left\{ \frac{1}{2} \left\| X - S \right\|_F^2 + \lambda \left\| X \right\|_* \right\}$$

Image | Reg. Parameter | Variable

$$\left\| A \right\|_F = \sqrt{trace\left(A \cdot A \right)} = \sqrt{\sum_{i=1}^{\min(m,n)} \sigma_i}$$

Approximation Term

$$\left\| A \right\|_* = trace\left(\sqrt{A \cdot A} \right) = \sum_{i=1}^{\min(m,n)} \sigma_i$$

Regularization Term

$A \in \Re^{m \times n}$

Image Processing Methods: Regularization

- Regularization problem:

$$\arg\min_{X}\left\{\frac{1}{2}\left\|X-S\right\|_F^2+\lambda\left\|X\right\|_*\right\}$$

$$\lambda>0$$

$$\lambda\downarrow:\ \left\|\bullet\right\|_F^2\downarrow\ \ und\ \ \left\|\bullet\right\|_*\uparrow$$

$$\lambda\uparrow:\ \left\|\bullet\right\|_F^2\uparrow\ \ und\ \ \left\|\bullet\right\|_*\downarrow$$

Singular Value Thresholding (SVT)

Soft-Thresholding:

$$T_\lambda(\sigma) = \begin{cases} \sigma - \lambda & for\ \sigma > \lambda \\ 0 & for\ \sigma \in [0, \lambda] \end{cases}$$

Hard-Thresholding:

$$T_a(\sigma) = \begin{cases} \sigma_i & for\ i \leq a \\ 0 & for\ i > a \end{cases}$$

Image Processing Methods: Regularization

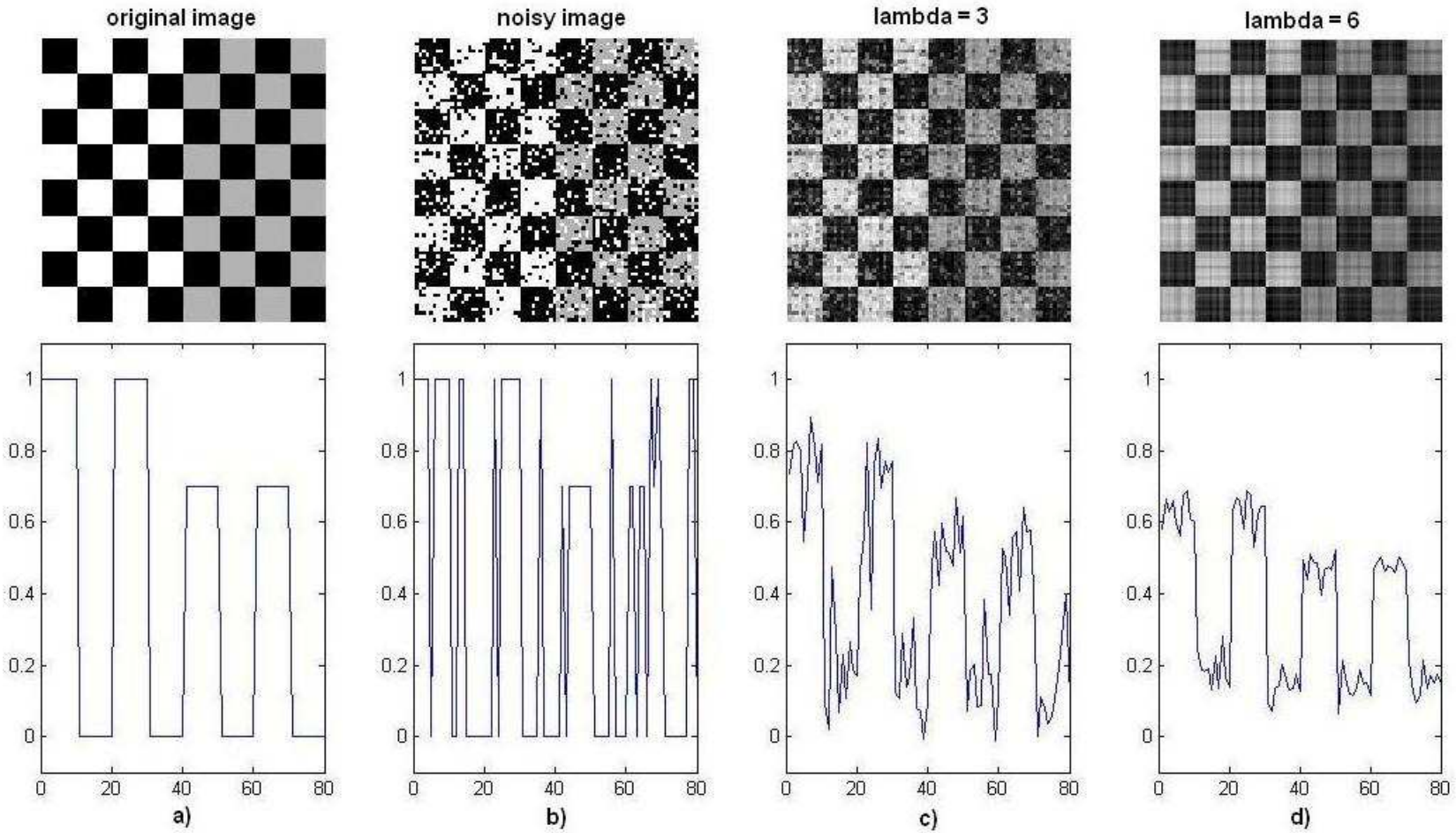
- application of $T_{\lambda/a}(\sigma)$

  is equivalent to minimizing $\left\{\frac{1}{2}\left\|X-S\right\|_F^2+\lambda\left\|X\right\|_*\right\}$

↵

$$U\,diag\left(T_{\lambda/a}(\sigma)\right)V^T \Leftrightarrow \arg\min_X\left\{\frac{1}{2}\left\|X-S\right\|_F^2+\lambda\left\|X\right\|_*\right\}$$

Soft-Thresholding



Hard-Thresholding

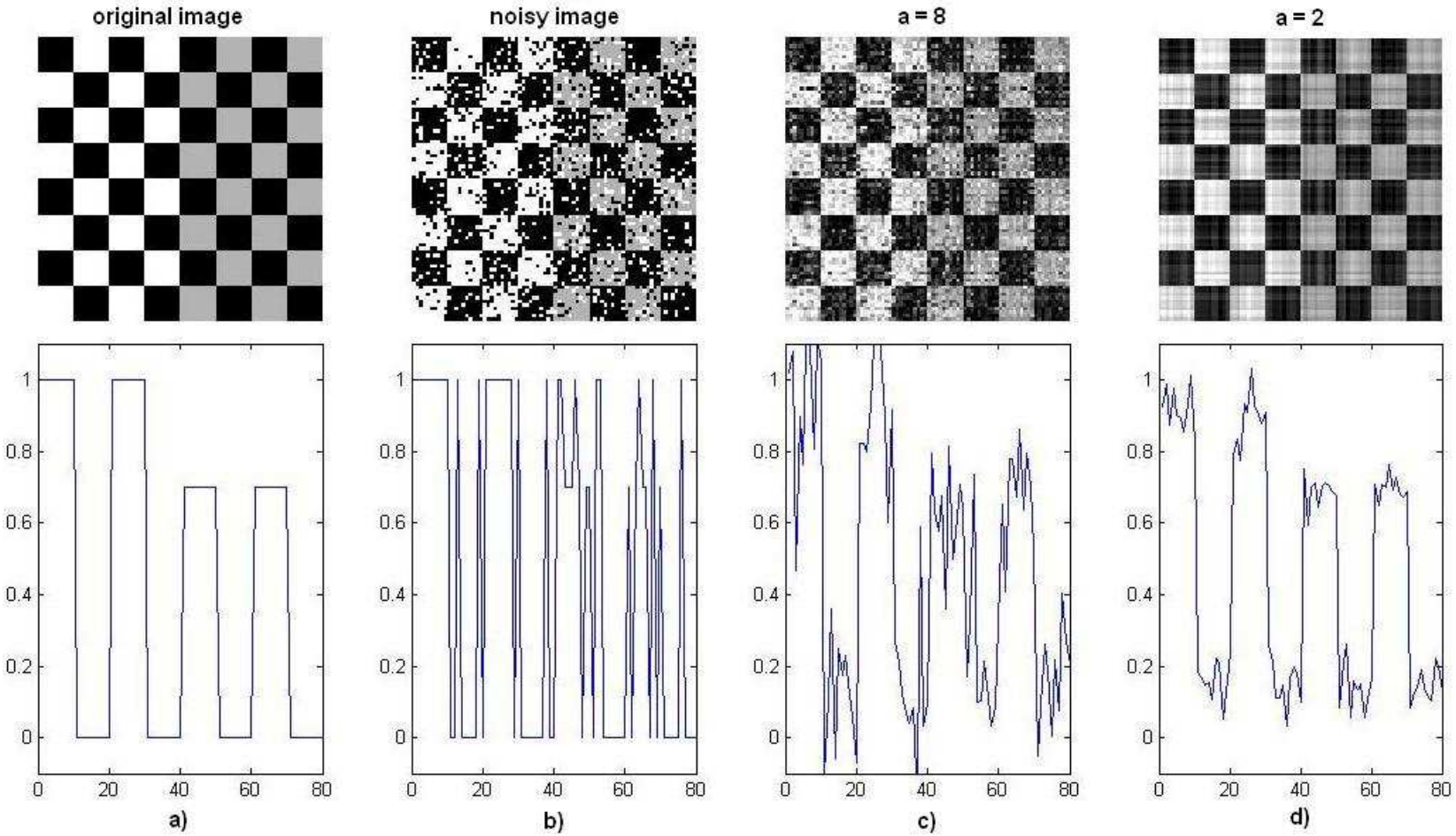


Image Processing Methods: Regularisation

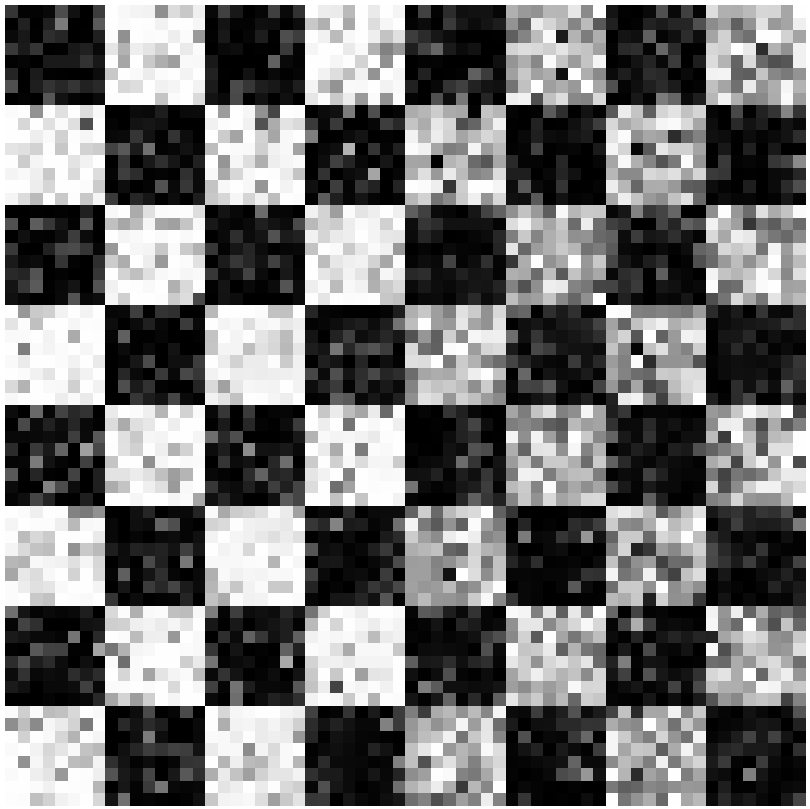
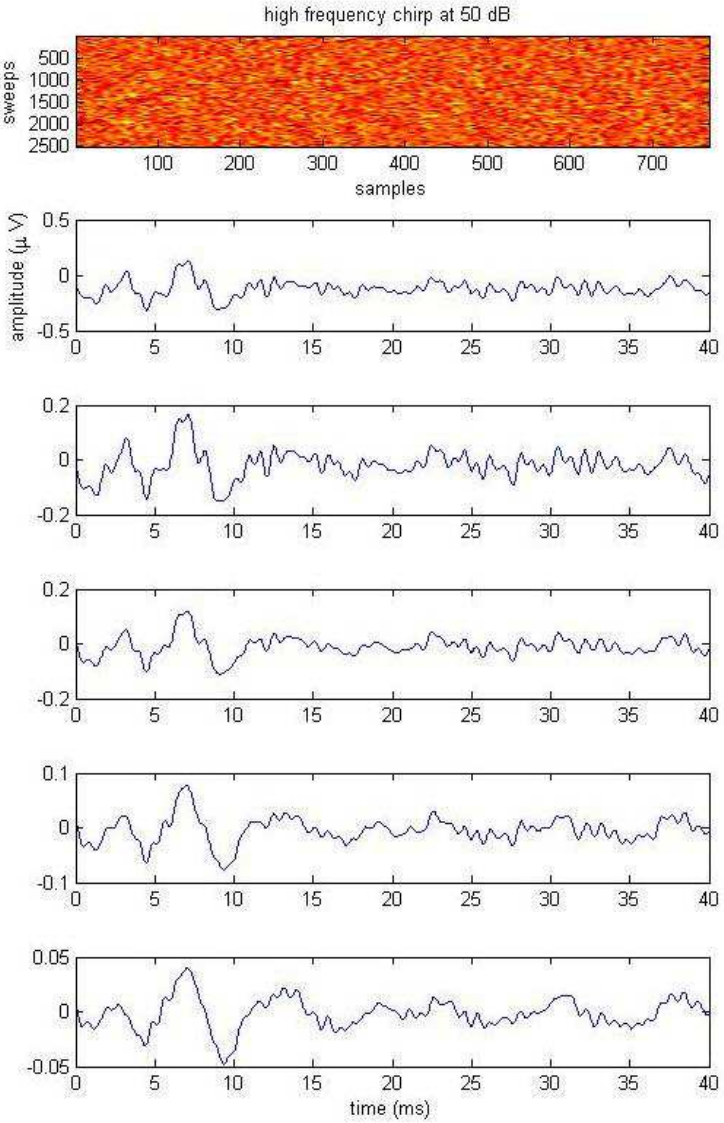
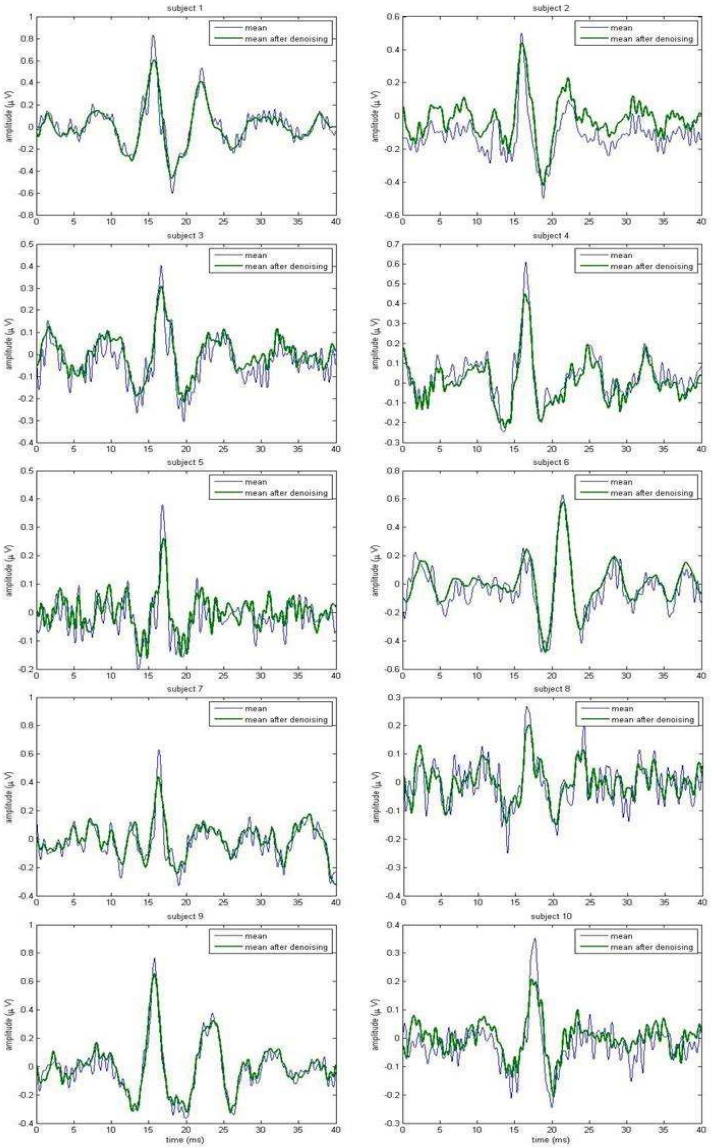


Image Processing Methods – Regularisation





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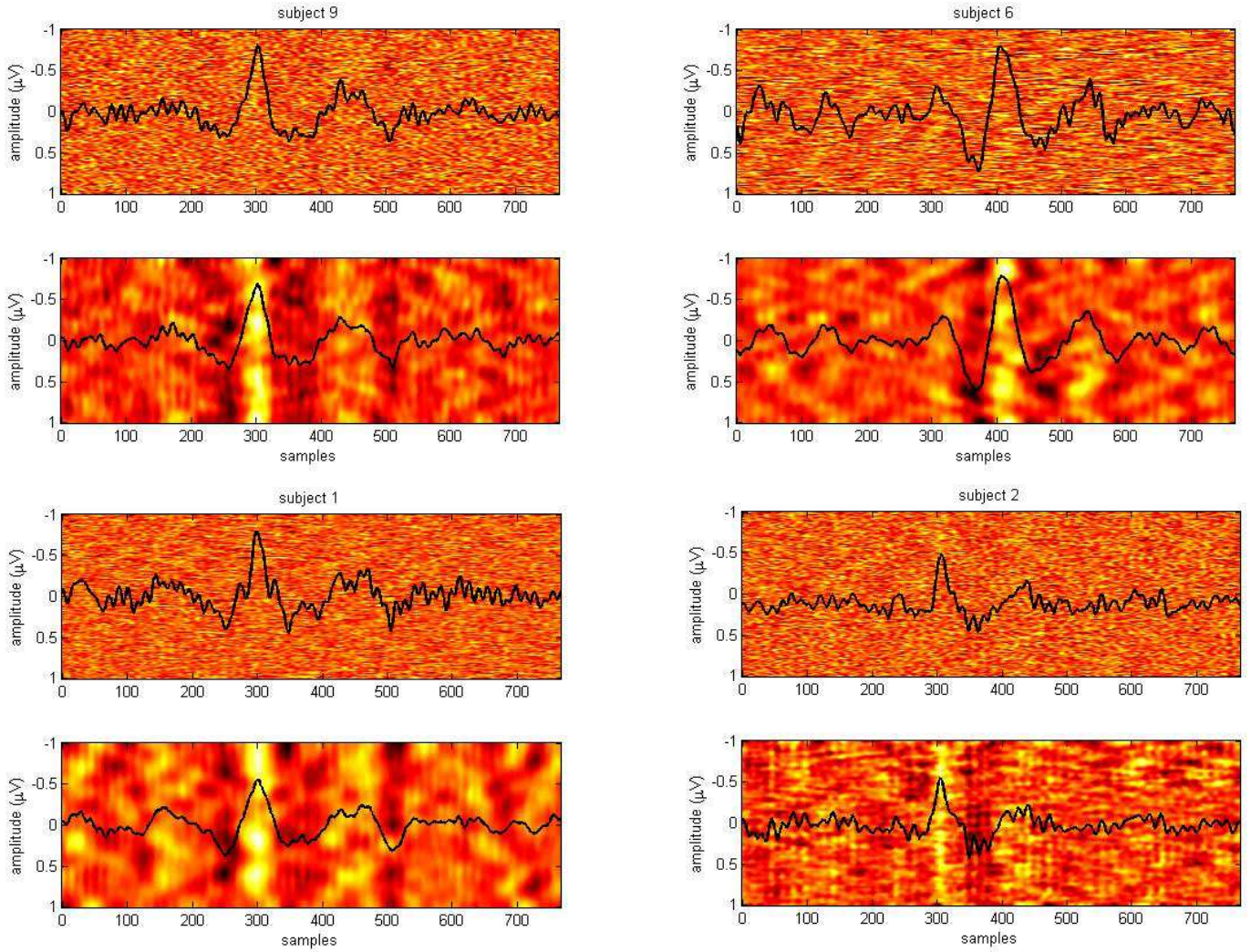
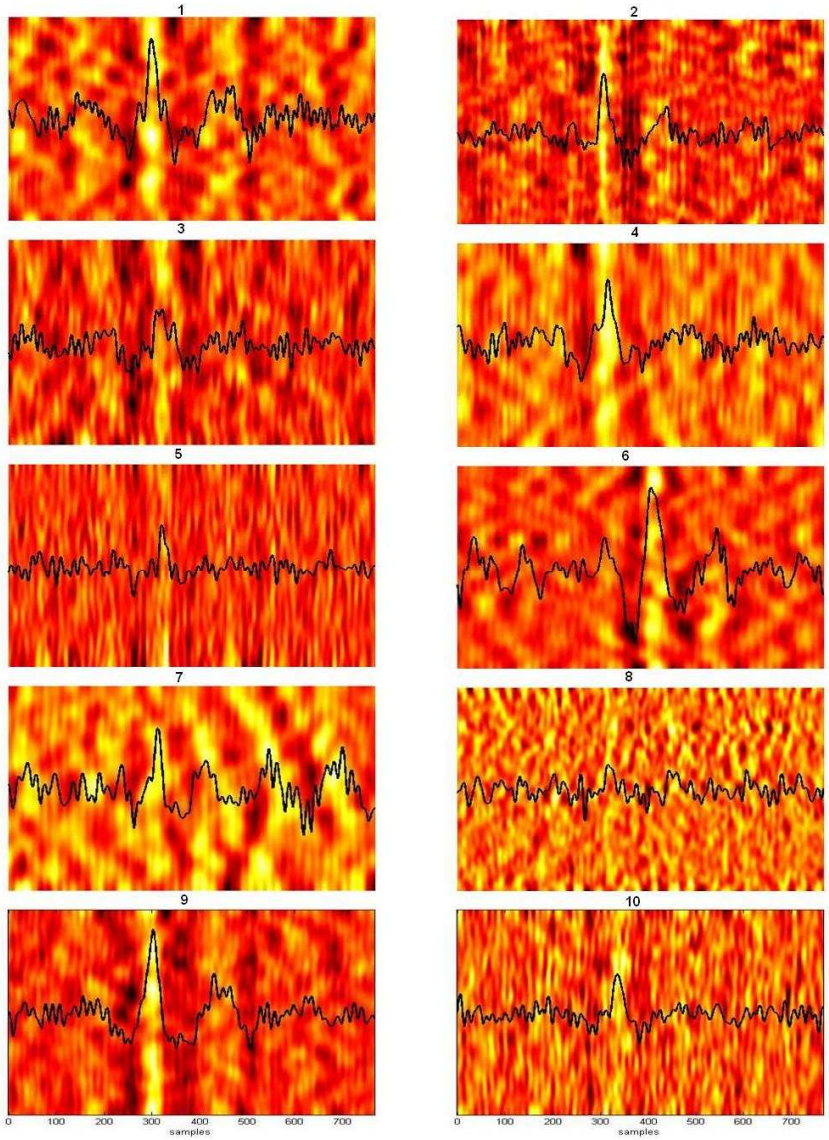
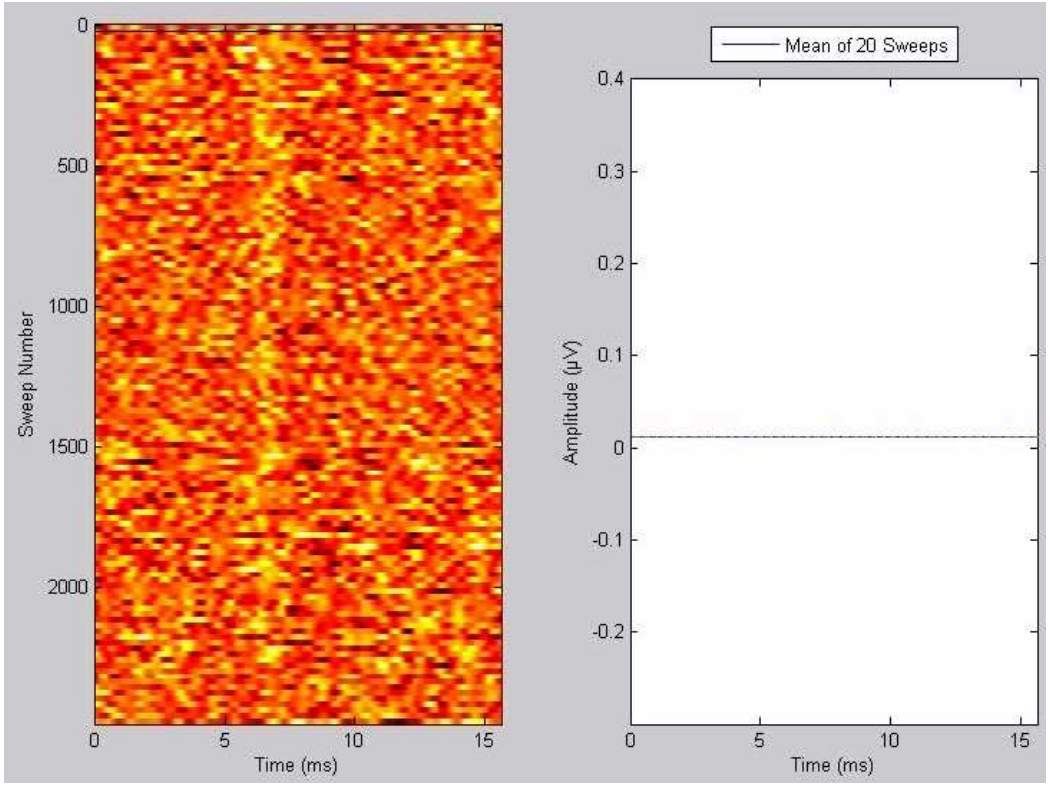


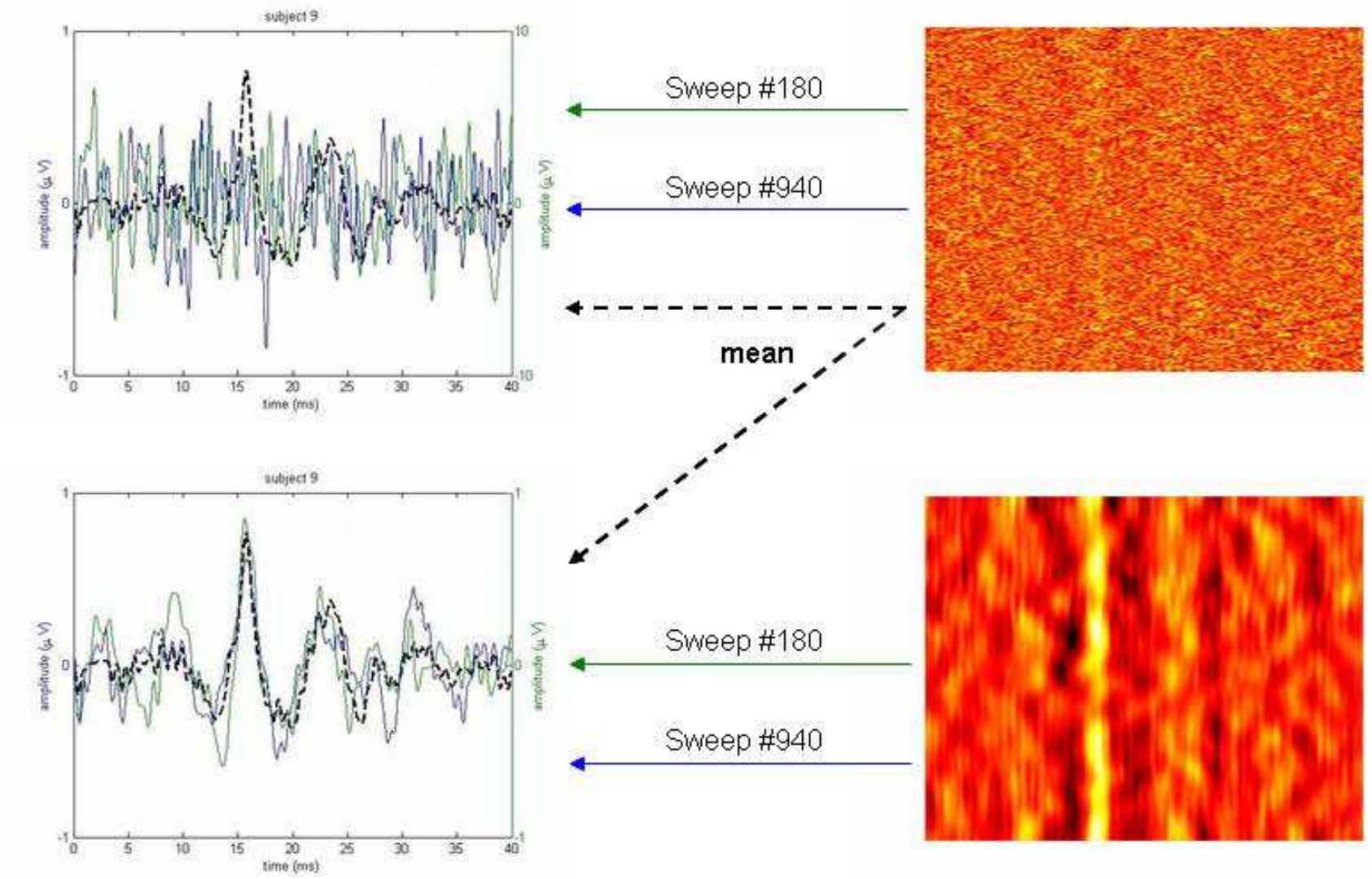
Image Processing Methods – Regularisation





averaging over the whole single-sweep matrix

Image Processing Methods – Regularisation



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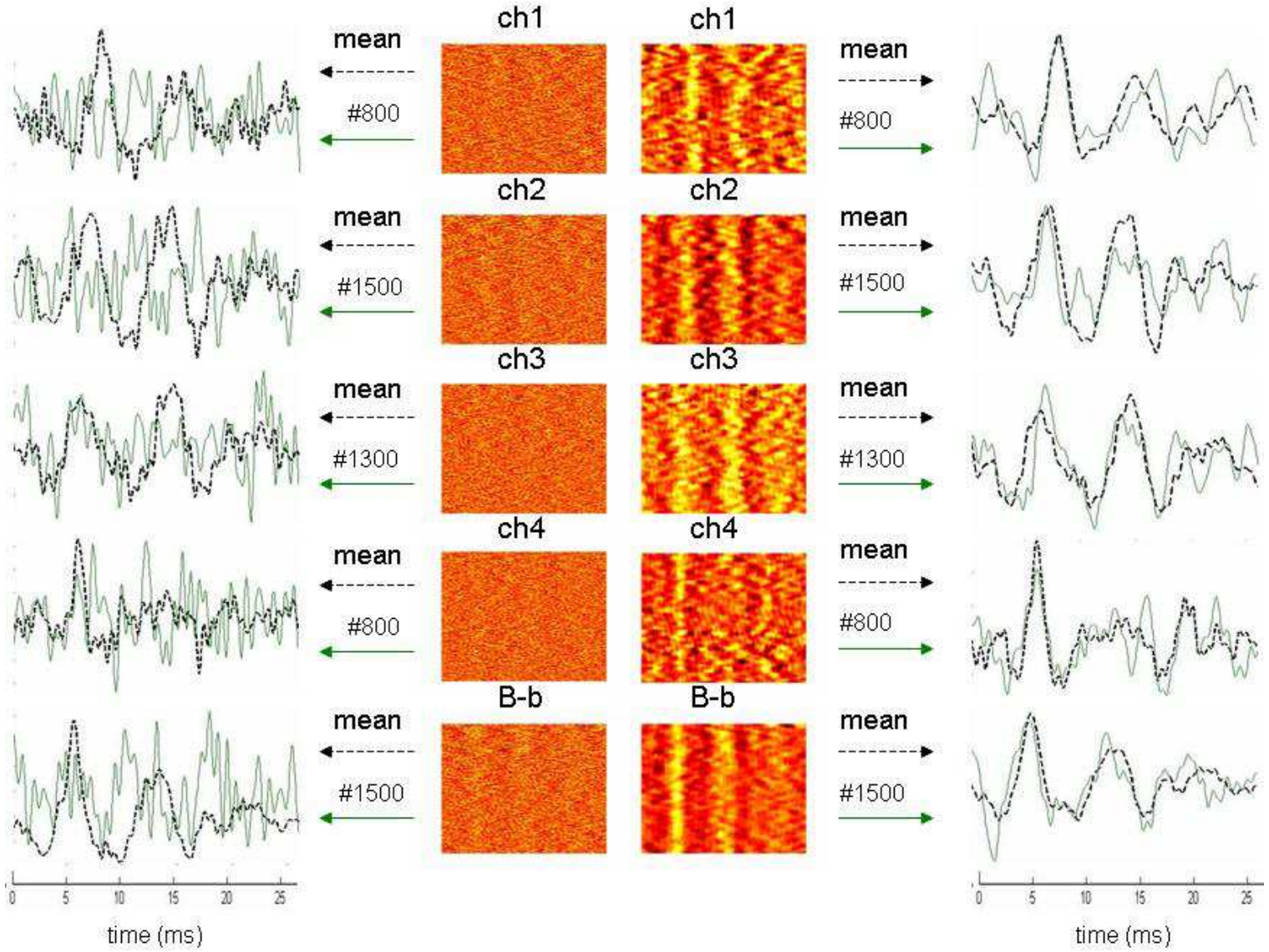

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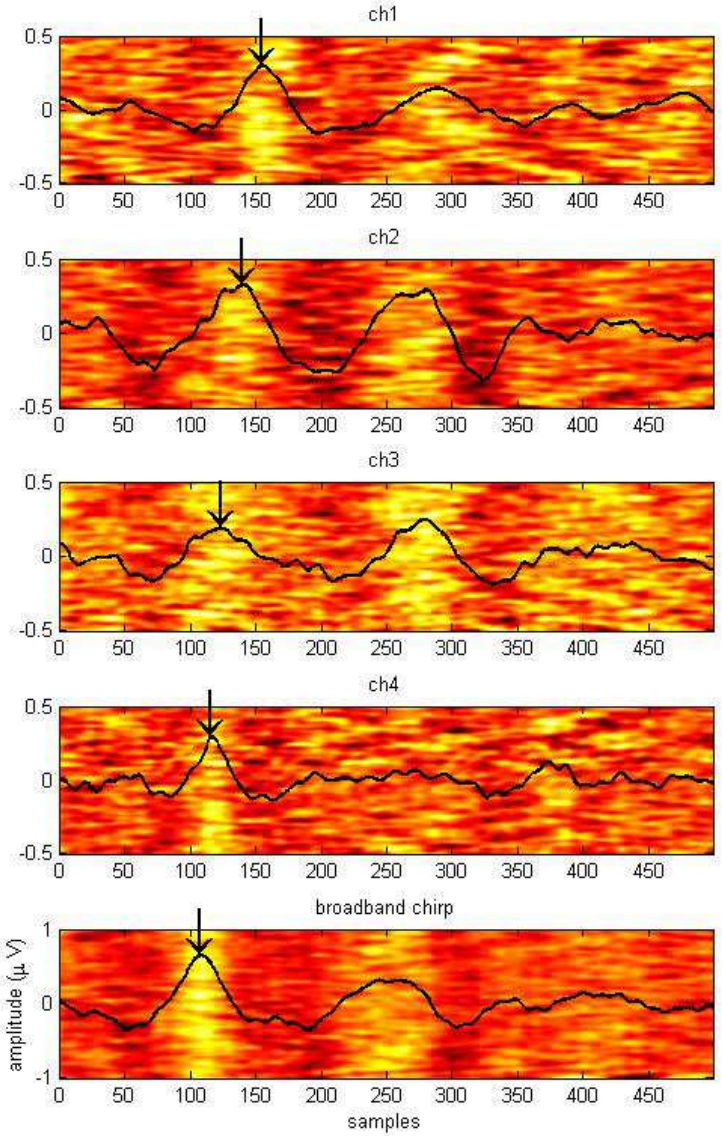


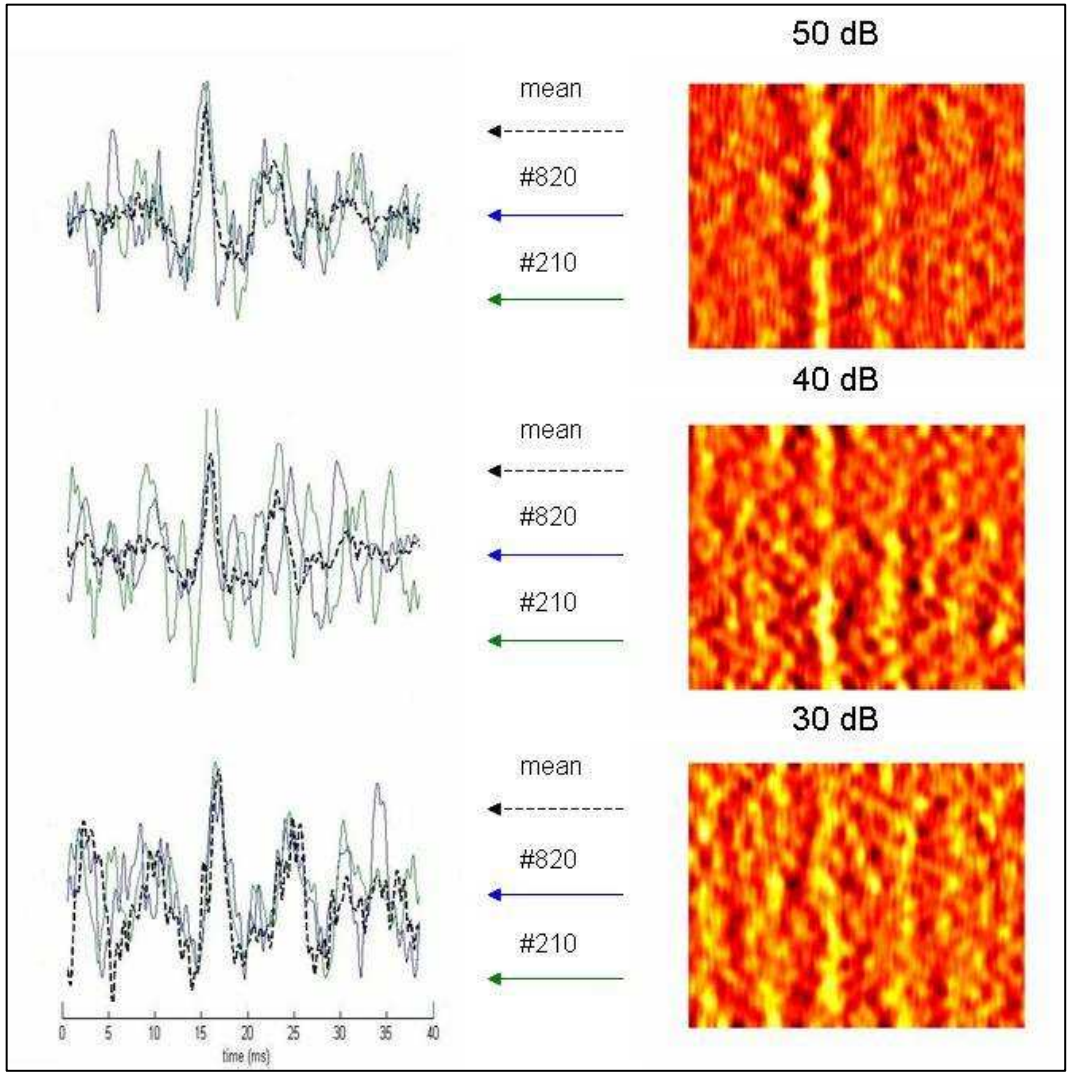


Image Processing Methods – Regularisation

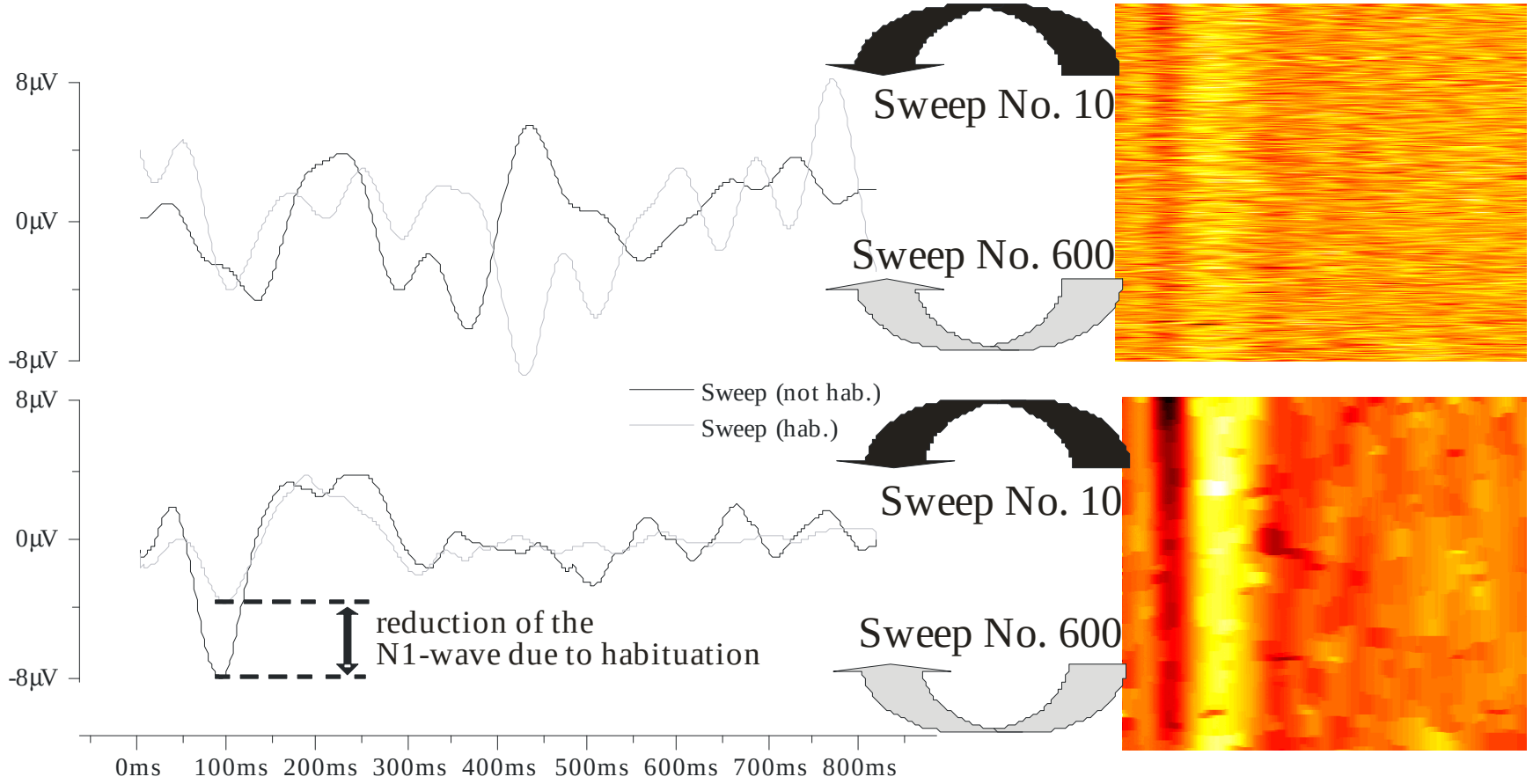
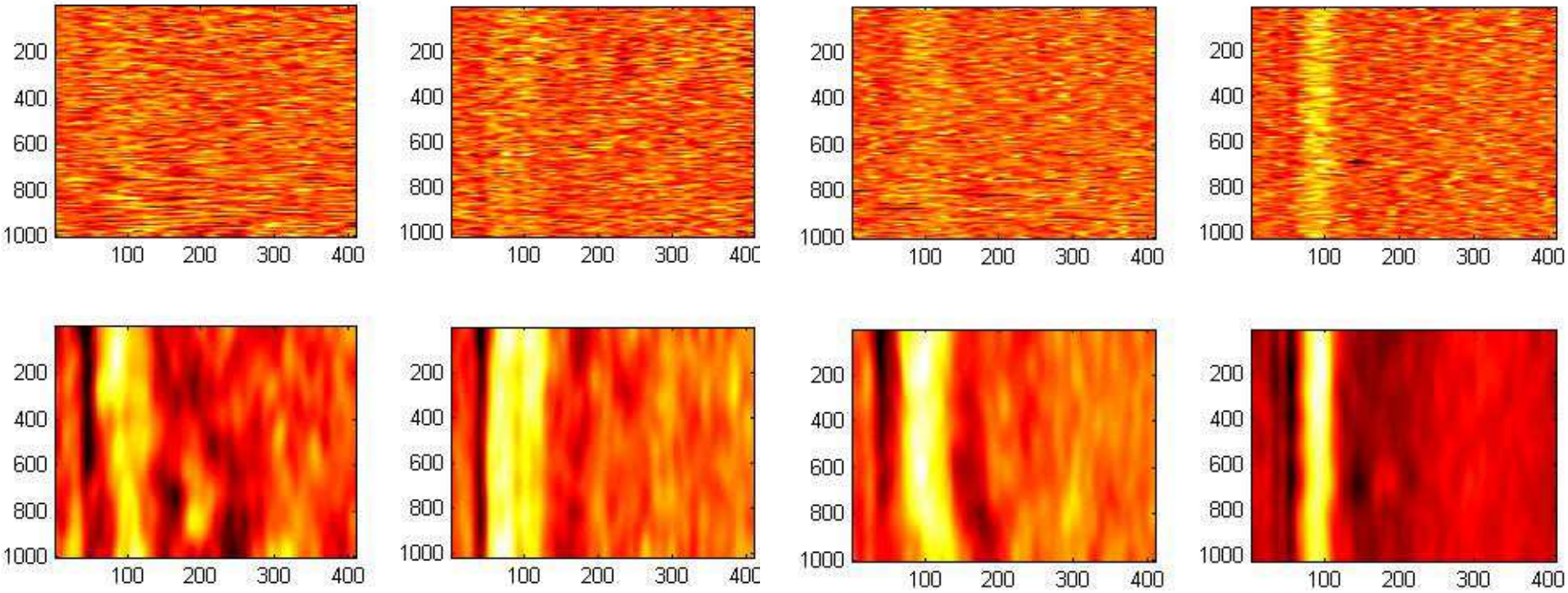
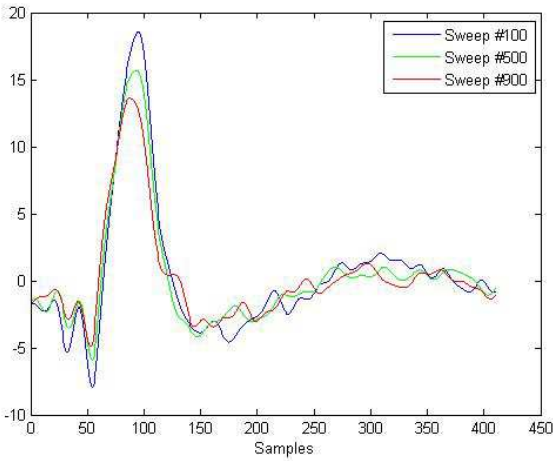


Image Processing Methods – Regularisation





Habituation

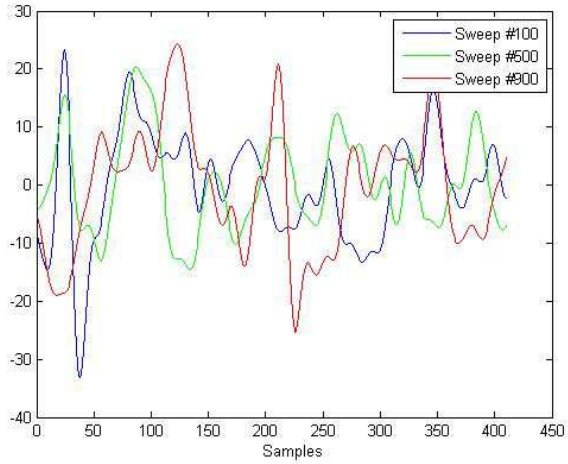
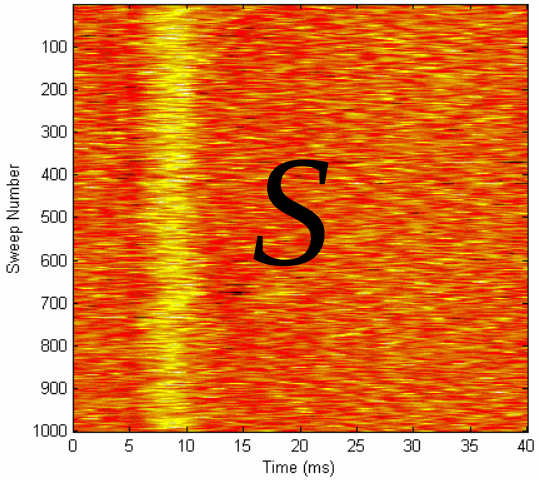
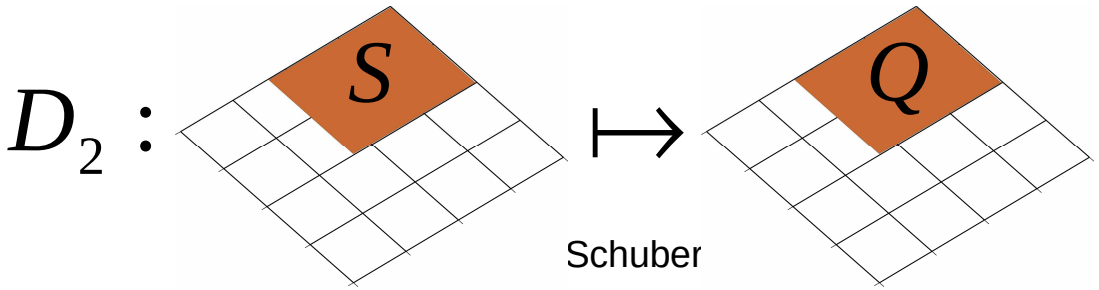


Image Processing Methods – Non-Local Means (NLM)

- ABR image $S \in \mathbb{R}^{N \times M}$ such that $S = \left(s_1, s_2, ..., s_N\right)^T$
- evoked individual sweeps s_i:
  - **time-locked to the stimulus**
  - **morphologically stable transient potential fluctuations**
  - **correlated intervals in the rows of**
  - → **induces self similarity in the ABRI**



- conventional denoising: $D_1 : s_n \mapsto \tilde{s}_n$, where $\tilde{s}_n \in \mathbb{R}^M \quad (n = 1, 2, ..., N)$
- new denoising operator: $D_2 : S \mapsto Q$, where $Q \in \mathbb{R}^{N \times M}$
- from $Q = \left(q_1, q_2, ..., q_N\right)^T$ the individual denoised sweeps $q_n \in \mathbb{R}^M$ can be extracted
- allows for an integration of regularities over multiple stimulations



Schuber

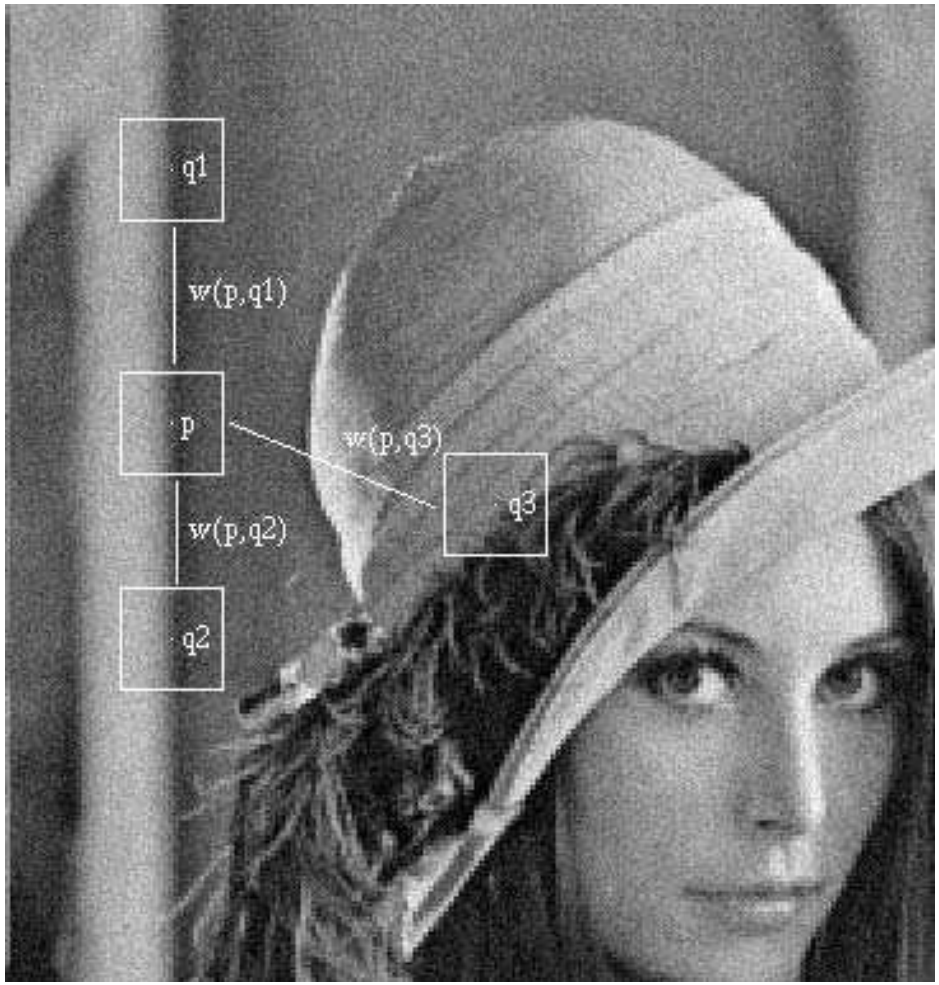
Image Processing Methods – Non-Local Means (NLM)

- exploits the self-similarity in images
- each sample (entry,pixel) s_i is replaced by its restored version q_i

$$q_i = \frac{1}{\gamma_i} \sum_{j=1}^{J} \xi_{i,j}\, s_j \qquad \gamma_i = \sum_{j=1}^{J} \xi_{i,j}$$

$$\xi_{i,j} = \exp\left(-\frac{1}{\lambda} \sum_{k \in I} \varphi_{\sigma,k} \left| s_{i+I} - s_{j+I} \right|^2 \right)$$

Image Processing Methods – Non-Local Means (NLM)



Scheme of NL-means strategy:

Similar pixel neighborhoods give a large weight, w(p,q1) and w(p,q2),

while much different neighborhoods give a small weight w(p,q3).

Image Processing Methods – Non-Local Means (NLM)

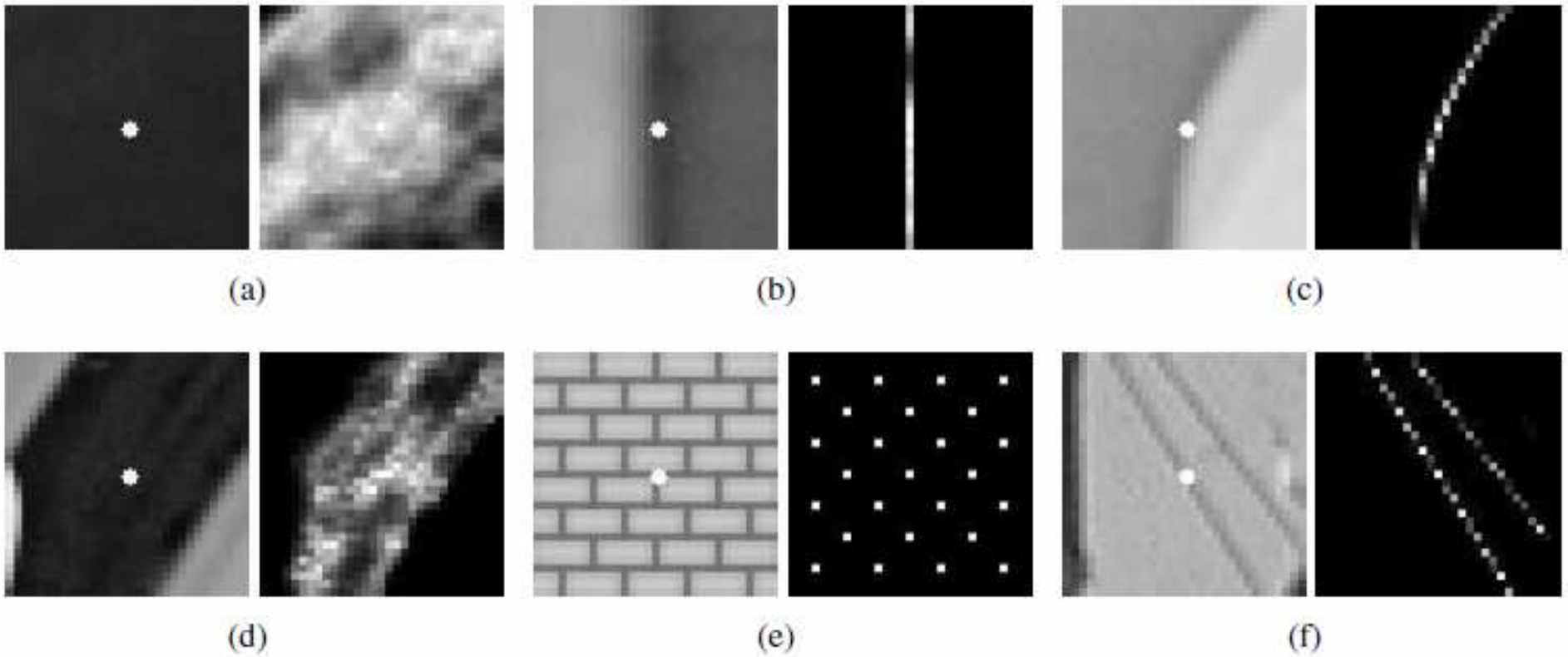


Figure 2. Display of the NL-means weight distribution used to estimate the central pixel of every image. The weights go from 1(white) to zero(black).

Image Processing Methods – Non-Local Means (NLM)

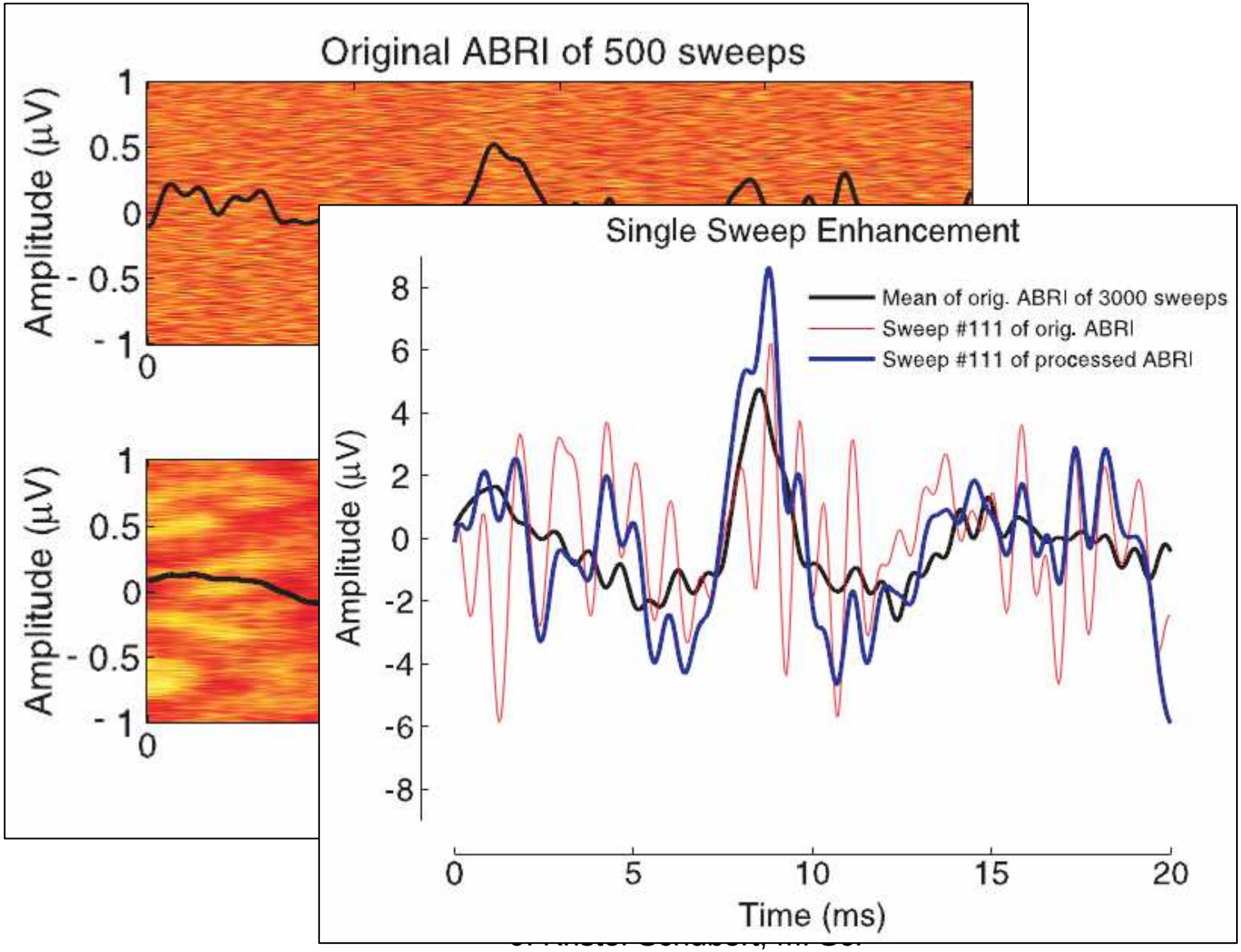
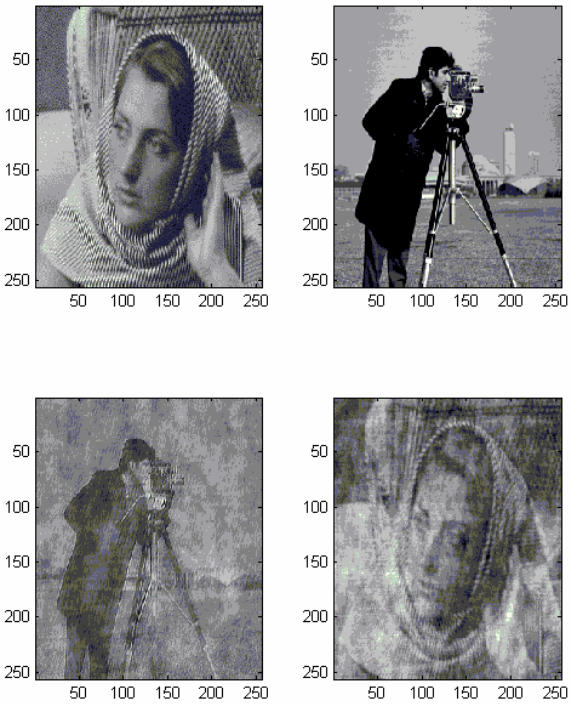
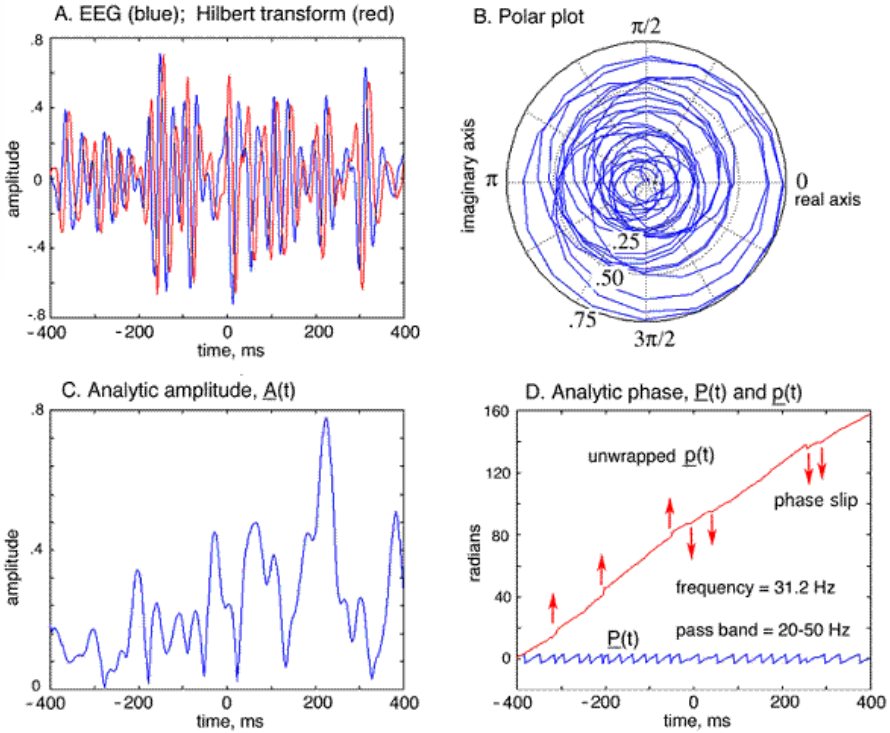


Image Processing: Phases of Images

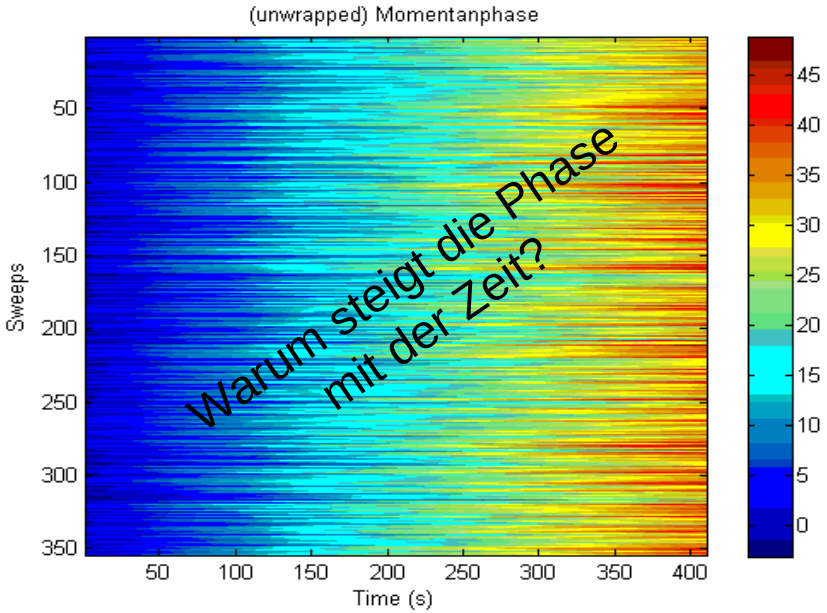
Why Phase? How to calculate the Phase?

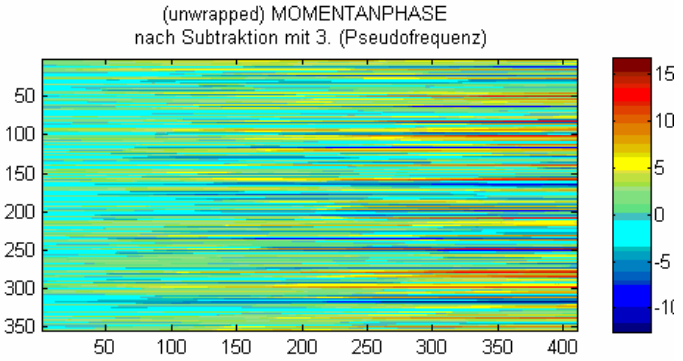


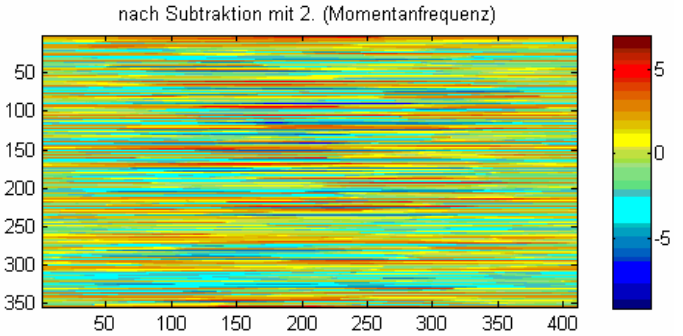


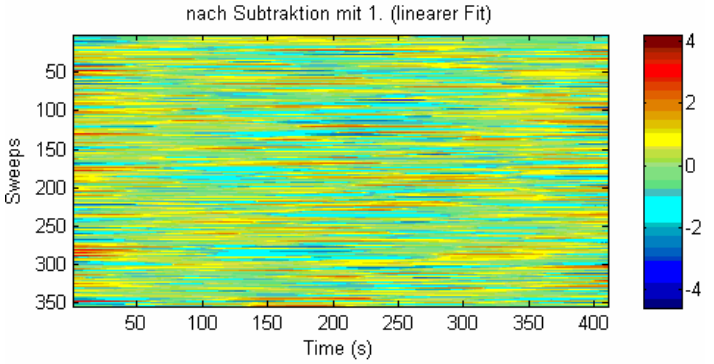
Fourier, Gabor, Wavelet, Hilbert,…

Phase







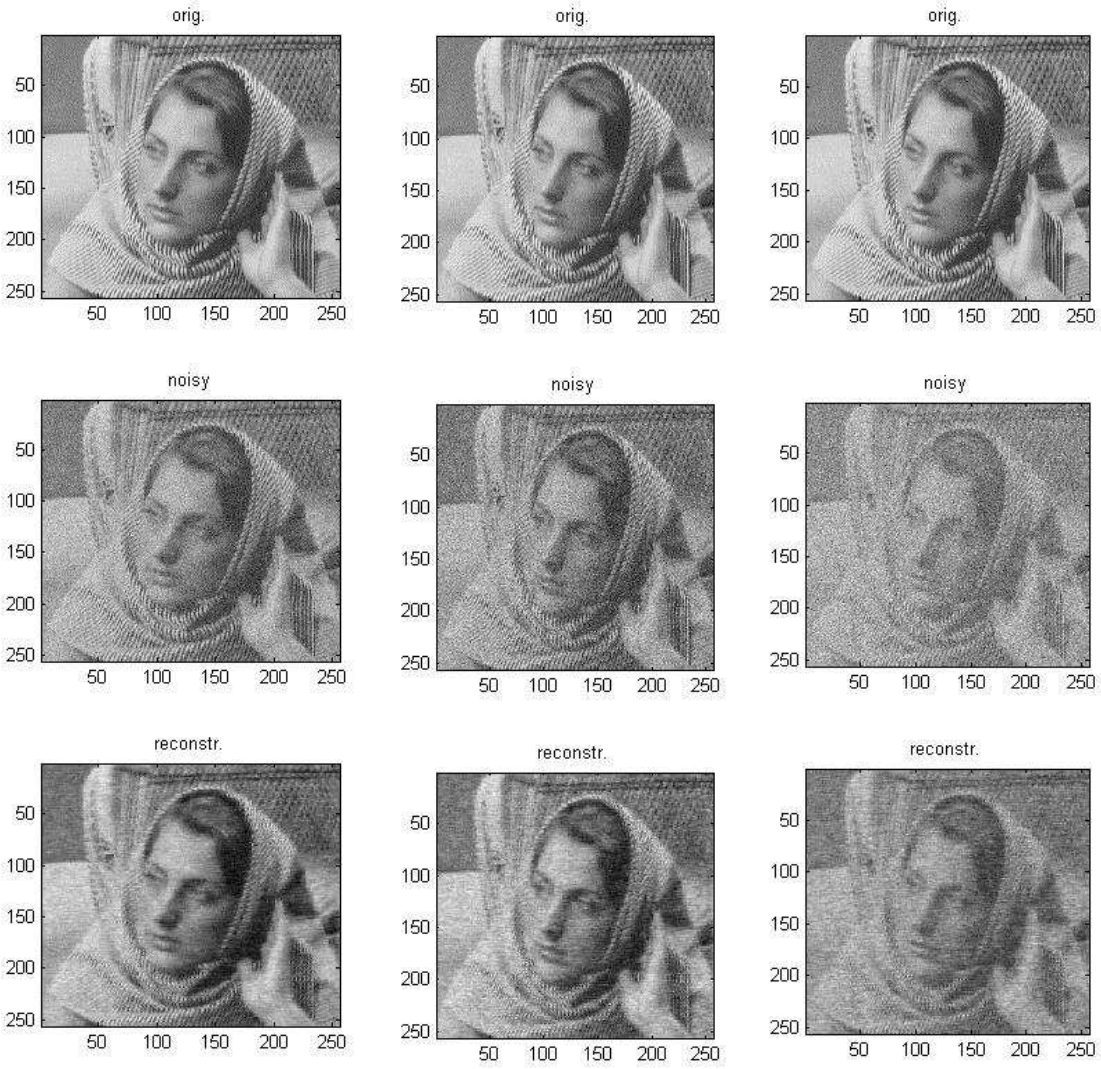


$$f = 8Hz$$

$$f_S = 512Hz \rightarrow T_S = {}^{1}\!/_{512Hz} \approx 0.002s$$

$$\Delta = \left(\frac{2\pi}{T}\right) \cdot T_S = \frac{(2\pi \cdot f)}{f_S} = 0.0982rad \ (\hat{=} 5.6°)$$

Phase Denoising



Obere Reihe:
Originales Bild (woman)

Mittlere Reihe:
Additives Rauschen (randn, Varianz = 0.005, 0.01, 0.03 von links nach rechts)

Untere Reihe:
Rekonstruiertes Bild. Hier wurde das Phasenbild nach Abzug einer gefitteten Geraden (Methode 4) mithilfe eines 2D Medianfilters (3x3) geglättet.

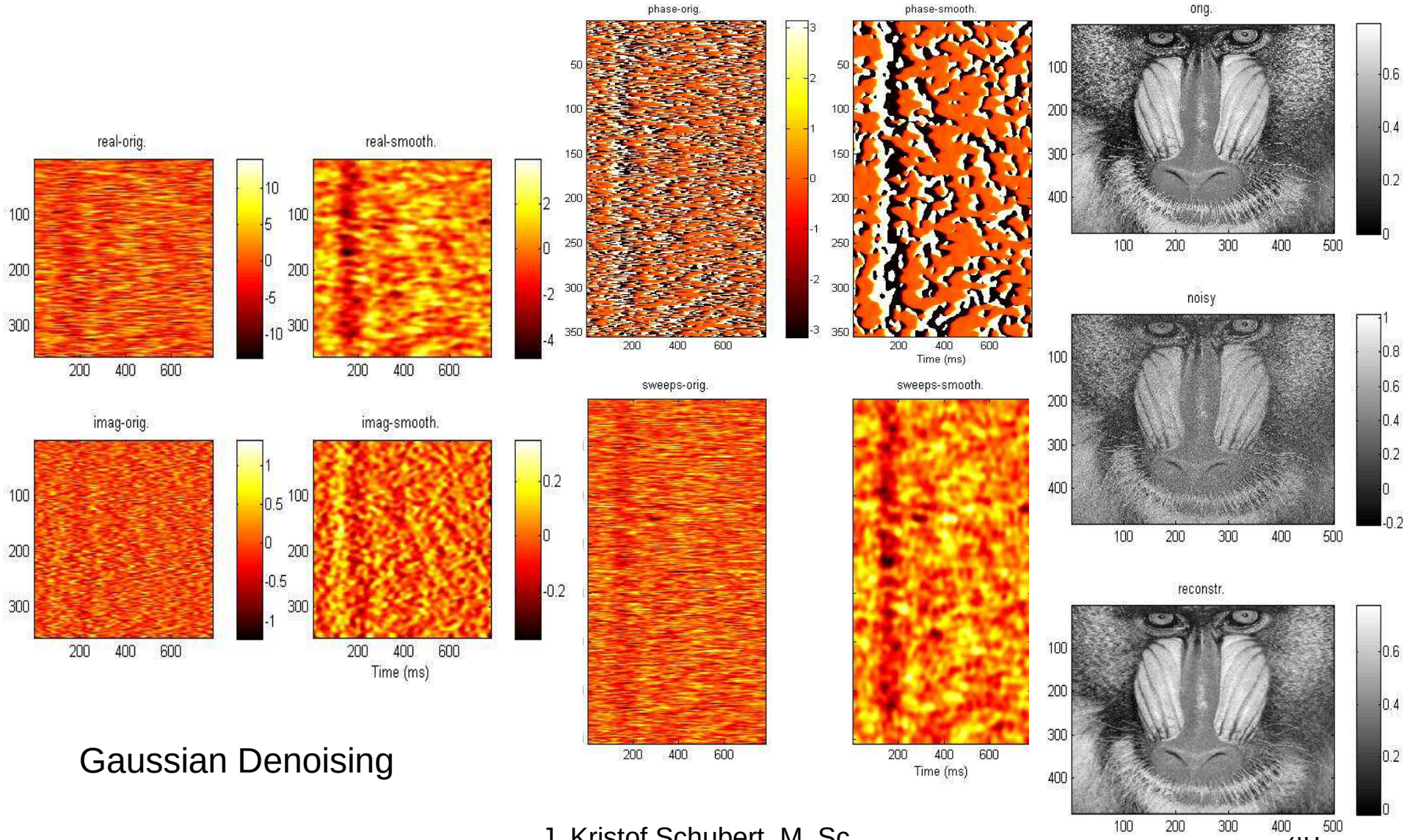
Phase Denoising

Obere Reihe:
Originales Bild (woman)

Mittlere Reihe:
Additives Rauschen (randn, Varianz = 0.005, 0.01, 0.03 von links nach rechts)

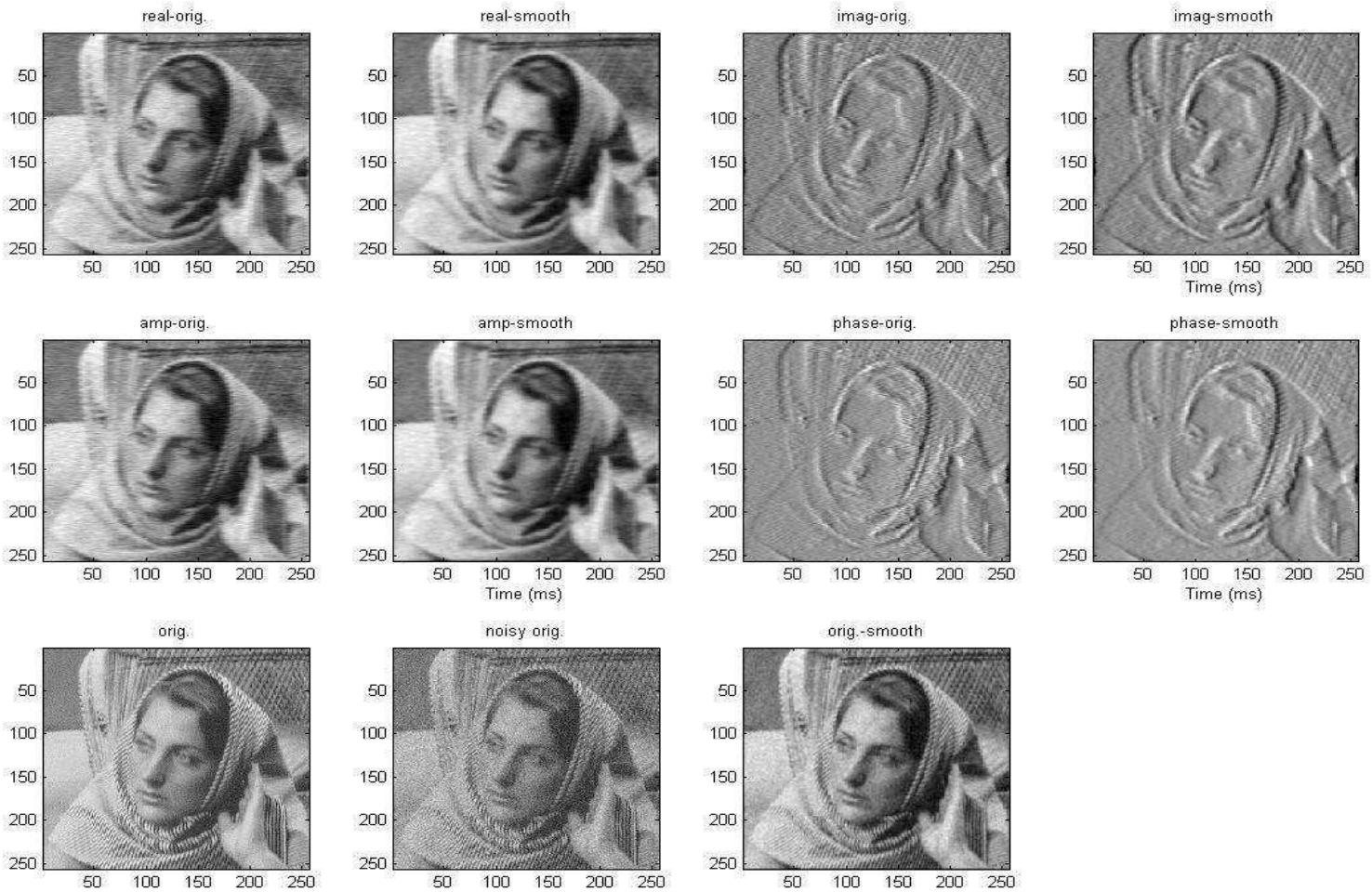
Untere Reihe:
Rekonstruiertes Bild. Hier wurde das Phasenbild nach Abzug einer gefitteten Geraden (Methode 4) mithilfe eines 2D Gaussian Blurrings (3x3, sigma=1) geglättet.

Phase Denoising

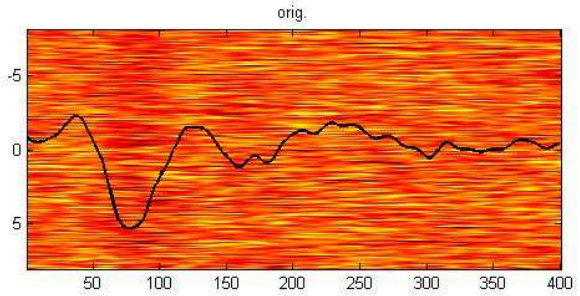


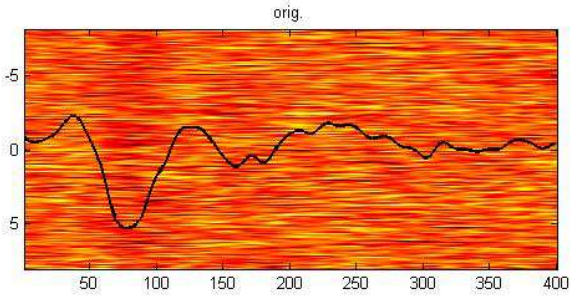
Gaussian Denoising

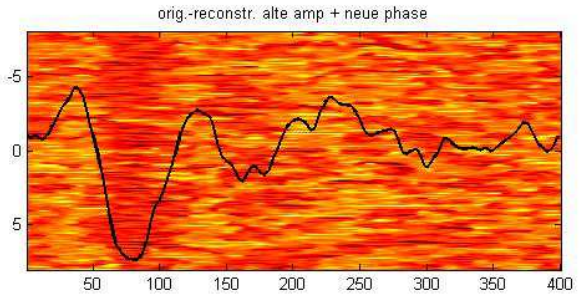
Phase Denoising

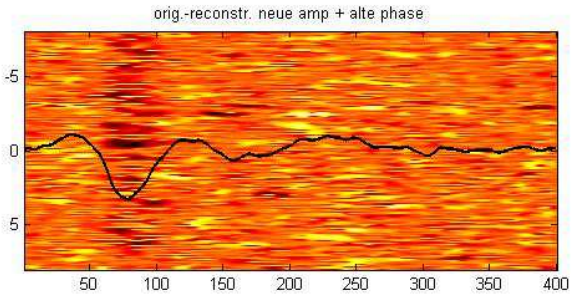


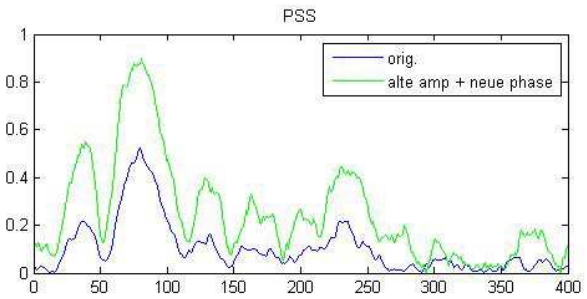
Phase Denoising

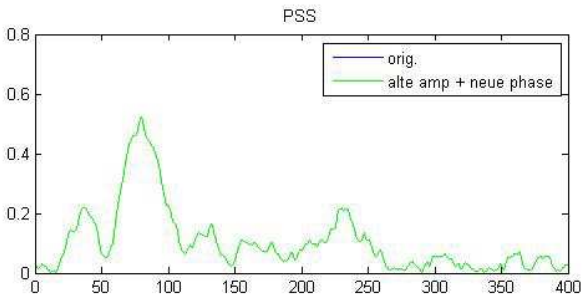






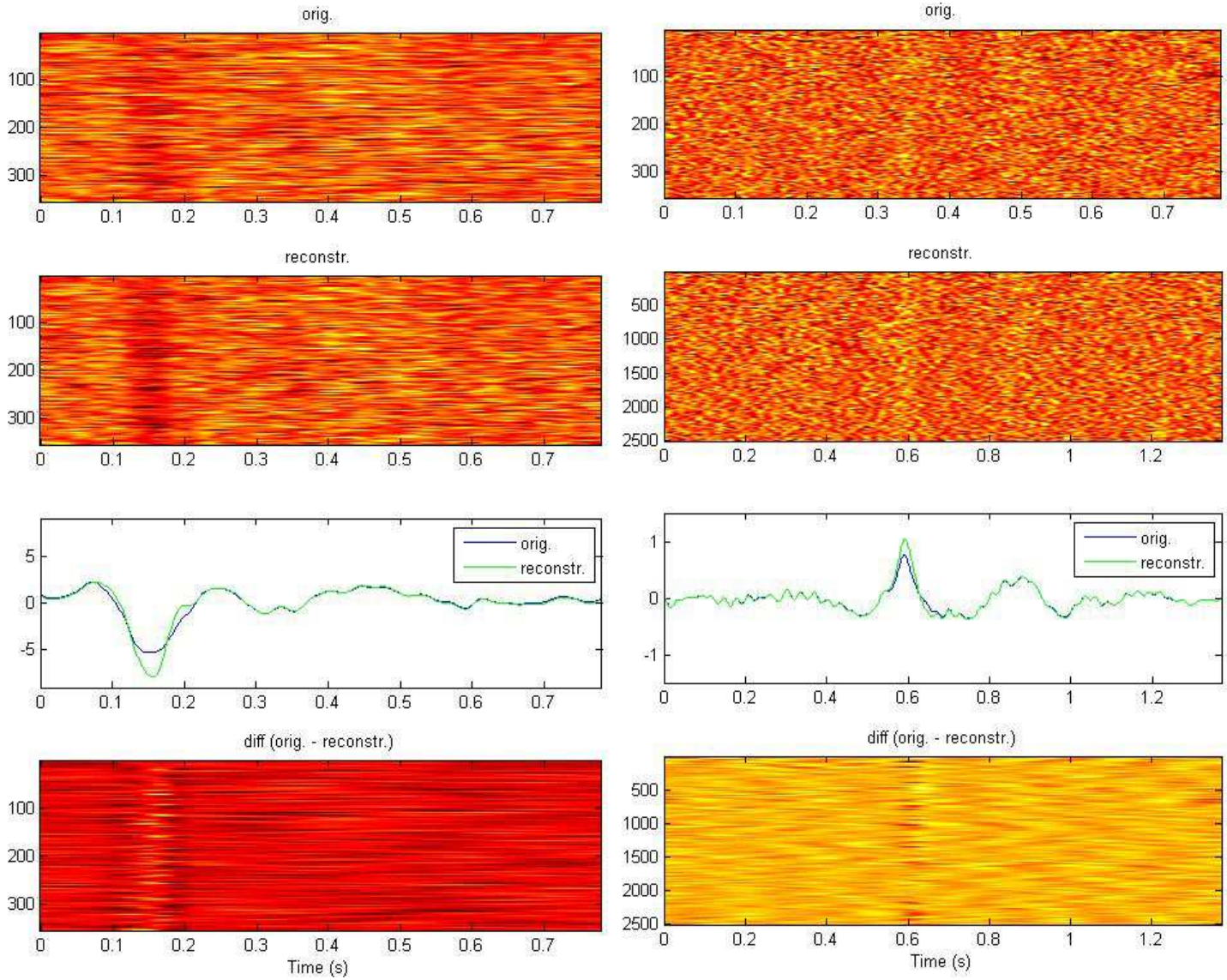






old vs. new phase

Phase Denoinsing



Smoothing via Thresholding the phase variance

Thank you for your attention!